Department of Economics
Simon Fraser University

Lucas M Herrenbrueck
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NUMERACY REVIEW FOR ECONOMICS STUDENTS

"Numeracy" means being comfortable with mathematics, just like "literacy" means being comfortable with language. Because mathematics is the language of nature, every science requires a high standard of numeracy, and economics is no exception. This review is supposed to help you find out if you meet that standard, and which topics, if necessary, you need to brush up on during the first week of classes. You should be able to do all of the exercises <u>quickly and without a calculator</u>.

1. Logarithms and exponentials

Many economic variables by definition cannot be negative and tend to grow (or shrink) proportionally instead of linearly. Such variables are well represented by exponential growth (or decay). The natural logarithm is the function that inverts the exponential, which makes it incredibly useful in such settings. (Note: the natural logarithm is the only one we will ever use, so $\log$ refers to that one. Some math textbooks write $\ln$ instead.) Its key properties are:

$$\log(xy) = \log(x) + \log(y)$$
$$\log(x^a) = a\log(x)$$
$$\log(1+x) \approx x \text{ (when x is a small number)}$$

These properties are so important because they show how the logarithm transforms products into sums and proportional growth to linear growth, and the latter are much, much easier to work with. To keep pace with an advanced economics course, these properties should be as familiar as basic arithmetic to you.

Exercise 1: Transform the equation $Y = K^{0.3}(AL)^{0.7}$ into logarithms.
Exercise 2: Compute $e^{0.03}$ and $\log(0.98)$ without a calculator.
Exercise 3: If X grows by 1% every year, in how many years will X double? You may use (and from here on memorize, please) the fact that $\log(2) \approx 0.7$.
Exercise 4: Look up what a *ratio scale* is and learn how to interpret it, and when to use it.

2. Percentages and proportions

Arithmetic like $\frac{1}{0.05} = 20$ (think: "twenty nickels make a dollar"), "one-fifth is more than one-tenth", and "percentages have to add up to one" should come easy to you. You should also know the difference between percentages and decimals (2% = 0.02, 0.8 = 80%, etc.), and percentages (abbreviated %) and percentage points (abbreviated *p.p.*).

Exercise 1: If variable X shrinks by 20% in the first year, then grows by 20% in the second year, how much has it changed overall?
Exercise 2: If variable X grows by 10% every year, how much will it grow in 3 years?
Exercise 3: Compute $\frac{1}{0.01}, \frac{1}{0.02}, \frac{1}{0.03}, \frac{1}{0.04}, \frac{1}{0.06}, \frac{1}{0.08}$, and $\frac{1}{0.09}$ approximately.
Exercise 4: Say that 20% of all patients who contract a particular type of cancer die (the "mortality rate"). Fortunately, you have discovered a treatment that reduces this mortality rate by 10%. What is the new mortality rate after treatment?
Exercise 5: In the example above, by how many percentage points has mortality fallen?
Exercise 6: In the example above, is reducing the mortality rate by X% the same as increasing the survival rate by X%? Does this answer change if we use *p.p.* instead of %?

3. Means and standard deviations

The concepts of *mean*, *median*, *variance*, and *standard deviation* should be familiar, and how they are related. You can easily look up the definitions online. Remember that means, medians, and standard deviations are measured on the same scale, but variances are not.

For many economic variables, such as income differences within a country or between countries, the distributions are "skewed" and the "rich" are much richer than the "poor" are poor – at least when we look at the numbers in a linear way! In such cases, the *geometric mean* (look it up!) and standard deviation may be more meaningful than the *arithmetic mean* (this is the usual one) and standard deviation. This is also a situation where the median may be more useful and intuitive than the mean.

Exercise 1: If an exam has a mean of 60 and a standard deviation of 15, how exceptional is a score of 50? How exceptional is a score of 90?
Exercise 2: If an exam has a mean of 60 and a standard deviation of 15, does that mean that the exam scores are normally distributed?
Exercise 3: Look up the general formula for the geometric mean. Show exactly how you can use the logarithm to transform it into an arithmetic mean.

4. Calculus

Single variable calculus is essential for advanced courses in economics, and multivariate calculus is frequently used, too. Fortunately, you will rarely need to compute any but the simplest integrals, and the derivatives become much easier with practice.

Exercise 1: Differentiate $\log(3x)$, $3\log(x)$, e^{3x}, and $x^{0.2}$ with respect to x.
Exercise 2: Differentiate $K^{0.3}(AL)^{0.7}$ first with respect to K, then with respect to L.
Exercise 3: Say both K and L are functions of time (while A is not). Take the derivative of $Y(t) = K(t)^{0.3}(AL(t))^{0.7}$ with respect to *t*. Then do the same with $\log(Y(t))$. Which one is easier to work with? Can you interpret your results in terms of proportional growth?