The Liquidity-Augmented Model of Macroeconomic Aggregates

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WEB APPENDIX

NOT FOR PUBLICATION

W.1 Extension: trading frictions in the PM

There are two reasons for introducing search frictions explictly into the goods market. One is the fact that we have already assumed that shoppers are anonymous and unable to commit to promises. This fits more naturally with the idea that shoppers meet with only a small number of firms, and trade bilaterally. The second reason is that search frictions give rise to market power (firms receive some of the gains from trade), and to mismatch (some shoppers do not trade). The result of these two things is to make the velocity of money in the goods market endogenous (or, at any rate, more flexible than it was in the main text; see Equation (12), which reflected the fact that at least outside of the liquidity trap, every dollar in the economy got spent in the goods market). For future empirical applications, this additional flexibility is likely to be important; and the reader may be also interested in a version of the LAMMA where firms have market power.

Suppose that there are N firms (where N is large), who are price takers in the factor market; they rent labor and capital at market prices w and r exactly as in the main text. However, they have the ability to post output prices, and shoppers face search frictions: they are subject to a lottery whereby they observe the price of n firms with probability ψ_n (Burdett and Judd, 1983). Draws are independent across shoppers, firms, and time. After observing their set of prices (or none, if $n = 0$), shoppers will choose to spend all their money at the firm with the lowest price. There is no recall of prices seen in previous periods.

Because there is a continuum of consumers and a finite number of firms, the law of large numbers applies and each firm can perfectly forecast demand for its product, conditional on the price it has set. (Hence, this set-up abstracts away from inventory or unemployment concerns.) Now, what is that demand? Each shopper has a certain amount of money to spend, and a constant marginal rate of substitution between money and goods (ϕ_t, as derived in Section 2.3). Since ϕ_t is the same for all shoppers, and independent of their money holdings, all shoppers follow the same optimal strategy: spend all of their money on the firm with the cheapest price available, unless that price exceeds $1/\phi_t$. In the latter case, spend nothing. Thus, for any firm charging a price below this reservation price, the intensive margin of demand is unit elastic. Based on this intensive margin alone, the best thing for a firm to do would be to set their price equal to the reservation price.

However, there is also the extensive margin to be considered. Suppose that the c.d.f. of posted prices is $F(p)$, and that it has no mass points; then, a firm charging price p' will almost surely sell to $a(F(p'))$ shoppers, where:

$$a(F) = \sum_{n=0}^{\infty} \psi_n n(1-F)^{n-1}$$

Therefore, writing $q_t \equiv A_t^{-1}\alpha^{-\alpha}(1-\alpha)^{\alpha-1}r_t^{\alpha}w_t^{1-\alpha}$ for the real unit cost of producing one unit of output, and writing M_t' for the amount of money held by all shoppers in the PM, nominal profits of a firm with price $p' \leq 1/\phi_t$ are equal to:

$$M_t'\left(1 - \frac{q_t}{\phi_t p'}\right) a(F(p'))$$

Burdett and Judd (1983) proved that – as long as both $\psi_1 > 0$ and $\psi_n > 0$ for some $n \geq 2$ – the only equilibrium of this price-setting game is endogenous price dispersion, where all firms post the price distribution $F(p)$ as a mixed strategy and make equal profits in expectation. The equilibrium $F(p)$ indeed has no mass points, and some firms do charge the reservation price ($F(p) < 1$ for $p < 1/\phi_t$). Furthermore, Herrenbrueck (2017) showed that the total amount of output purchased equals:

$$Y_t = \left(\psi_0 \cdot 0 + \psi_1 \cdot 1 + (1-\psi_0-\psi_1)\frac{1}{q_t}\right) \cdot \phi_t M_t'$$

In words: a fraction ψ_0 of shoppers is mismatched and does not purchase anything (although they still get to hold on to their money). The rest of the solution is surprisingly simple: even though almost all shoppers spend a price in between the efficient price (q_t/ϕ_t) and their reservation price ($1/\phi_t$), the equilibrium is *as if* a fraction ψ_1 of them spent the reservation price and everyone remaining (who was matched with $n \geq 2$ firms) spent the efficient price, equal to marginal cost.

Since the marginal disutility of a dollar of spending is ϕ_t (the output good in the CM is the numéraire), and allowing for shocks to the matching parameters ψ_n, the Euler equations representing asset demands change as follows from Equations (6)-(8). First, define the ex-post liquidity premium ℓ_{t+1}:

$$\ell_{t+1} \equiv \lambda_{t+1}s_{t+1}^B\left(\psi_{0,t+1} + \psi_{1,t+1} + \frac{1-\psi_{0,t+1}-\psi_{1,t+1}}{q_{t+1}}\right) - \lambda_{t+1}$$

Then:

$$u'(c_t)\phi_t = \beta\,\mathbb{E}_t\left\{u'(c_{t+1})\phi_{t+1}\,\frac{1+\ell_{t+1}}{s_{t+1}^B}\right\}$$

$$u'(c_t)\phi_t p_t^B = \beta\,\mathbb{E}_t\left\{u'(c_{t+1})\phi_{t+1}\left(1+\ell_{t+1}\right)\right\}$$

$$u'(c_t) = \beta\,\mathbb{E}_t\left\{u'(c_{t+1})(r_{t+1}+1-\delta)\left(1+\eta_{t+1}\ell_{t+1}\right)\right\}$$

Note that if $\psi_0 = \psi_1 = 0$, i.e., all shoppers see at least two prices, then PM trade is *effectively* competitive and the Euler equations are the same as before. And, more precisely, these Euler equations hold under the assumption that the number of prices a shopper observes (n) is only revealed *after* the AM subperiod has concluded. If this was revealed at the beginning of a period, then shoppers with low n or high observed prices would choose to use their money to buy assets rather than goods in that period (Chen, 2015).

In steady state, and using the two interest rates $1+j \equiv 1/s^B$ and $1+i \equiv (1+j)(1+\ell)$ again, we obtain the following expression for the Friedman wedge:

$$q = \frac{1-\psi_0-\psi_1}{\frac{1+i}{1+\ell}\left(1+\frac{\ell}{\lambda}\right)-\psi_0-\psi_1}$$

The equations for the Mundell-Tobin wedge and for PM clearing (Equation 2) stay the same. Hence, the resulting monetary wedge follows the same formula as before:

$$\Omega(i,\ell) = q\cdot\frac{\rho+\delta}{r},$$

the only difference being that q now incorporates the matching friction terms ψ_0 and ψ_1. Their effect will be to push the wedge Ω down compared to the main text. First, if $\psi_1 > 0$, then firms have market power and shoppers will give up some surplus. Second, if $\psi_0 > 0$, then there is mismatch, and some shoppers will not be able to make a purchase. However, at the Friedman rule, $i = \ell = 0$ implies $q = 1$, as before. Thus, the effect of matching frictions in the goods market is to *rotate* the cone of policy options downwards around the origin – see Figure W.1 for an illustration. The result of this is that for any given policy interest rate, the inflation tax bites more keenly, and output is lower; equivalently, for any given inflation rate the optimal policy interest rate has to be lower.[1]

On the income side, how does the money held by shoppers get distributed after the PM? First, a fraction ψ_0 is unspent by the shoppers, hence they keep it. It can be shown (Herrenbrueck, 2017) that a fraction $(1-q)\psi_1/(1-\psi_0)$ of the remainder – or, equivalently, $(1-q)\psi_1$ of the total – goes to the owners of the firms as profits. The rest gets paid to the owners of factor inputs. Hence, a fraction $\alpha[1-\psi_0-(1-q)\psi_1]$ of the shoppers' money holdings goes to capital owners, and a fraction $(1-\alpha)[1-\psi_0-(1-q)\psi_1]$ goes to workers.

It is now straightforward to take the limit $(\psi_0,\psi_1)\to 0$, meaning that every shopper sees

[1] Unless we are in the "reversal" region of the parameter space, in which case the second-best level of the policy rate, for a given $i > 0$, is maximal – as before.

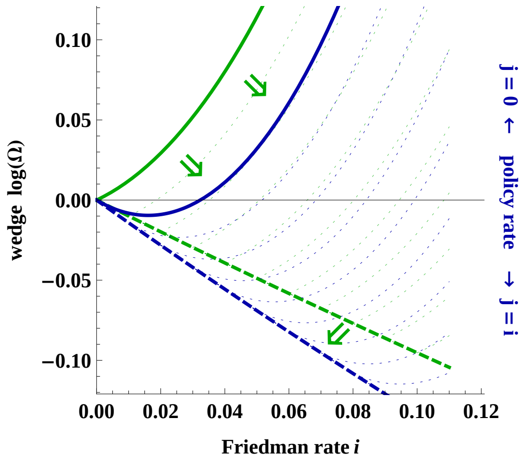


Figure W.1: Menu of policy options without (blue) and with (green) goods market frictions. Maintained parameters: $\rho = 0.03$, $\delta = 0.1$, $\lambda = 0.2$, $\eta = 0.75$. Varying: $(\psi_0 + \psi_1) \in \{0, 0.3\}$.

the prices of *at least two* firms. Then, the equations of the frictional model become identical to those of the competitive model in the main text.

Once firms make profits, it may be interesting to model firm equity explicitly. In particular, equity might be considered an indirectly liquid asset that can be sold in the AM, in the same way that capital is (see also Rocheteau and Rodriguez-Lopez, 2014). Other details can be added. For example, there may be firm entry subject to a cost, and entry by more firms may have the effect to improve the matching probabilities by shoppers in the sense of a FOSD shift in the distribution $\{\psi\}$ (see also Herrenbrueck, 2017).

W.2 Extension: interaction between fiscal and monetary policy

In this section, we split the consolidated government into a fiscal authority (in charge of bond issuance, B) and a monetary authority (in charge of the money supply, M), and analyze these authorities' policy options separately. We will not take a deeper look into "fiscal policy", which could also include cyclical policy, government spending on a public good, or distortionary taxation. All of these issues are also important, of course.

Suppose that the fiscal authority controls the sequence of bond issues, $\{B_t^F\}_{t=1}^{\infty}$, and it seeks to finance a sequence of nominal lump-sum transfers $\{T_t\}_{t=0}^{\infty}$ (taxes if negative). The fiscal authority is only active during the CM.

The monetary authority is active during the AM and the CM, and it controls the sequence of money supplies, $\{M_t\}_{t=1}^{\infty}$. It can intervene in the AM by buying up bonds with money, or selling bonds for money; denote the monetary authority's bond holdings at the beginning of period t by B_t^M, and assume that the monetary authority is not able to sell more bonds than it has: $B_t^M \geq 0$. Since the bond here is a one-period discount bond, this means that if the monetary authority wishes to be able to conduct an open-market sale in period $t+1$, it must buy some newly issued bonds in the CM of period t.

With this choice of notation, B_t^F indicates bonds *issued* by the fiscal authority, whereas B_t^M indicates bonds *held* by the monetary authority. The stock of bonds held by the public, at the beginning of period t, will then be $B_t \equiv B_t^F - B_t^M$.

At the end of a period, the monetary authority makes a seigniorage transfer to the fiscal authority, S_t. Here, we do not take a stand on whether this transfer can be negative as well as positive, or whether the monetary authority has authority over choosing its level. For example, it may be realistic to assume that the monetary authority has full authority over choosing *positive* levels of S_t, but requires the cooperation of the fiscal authority if it wants to collect a tax. Alternatively, a monetary authority that would like to increase inflation may have limited power if a more hawkish fiscal authority refuses to increase its spending; arguably, this has been the case in the Eurozone in recent years (Bützer, 2017).

Since the fiscal authority issues bonds in the primary market (the CM), where the bond price is p^B, it must obey the following budget constraint, for all $t \geq 0$:

$$p_t^B B_{t+1}^F + S_t = B_t^F + T_t,$$

along with the no-Ponzi condition that B_t^F / M_t remains bounded. The monetary authority must obey the following budget constraint, for all $t \geq 0$:

$$M_{t+1} - M_t + B_t^M = S_t + s_t^B (B_{t+1}^M - B_t^M) + p_t^B B_{t+1}^M$$

We can interpret this constraint as follows. On the left hand side is the 'revenue' of the monetary authority in period t: newly created money ($M_{t+1} - M_t$) and payments from redemption of the bonds in its portfolio (B_t^M). On the right hand side are the things this revenue can be spent on: the seigniorage transfer to the fiscal authority (S_t), open-market purchases of bonds from the public ($s_t^B (B_{t+1}^M - B_t^M)$), and purchases of newly issued bonds from the fiscal authority ($p_t^B B_{t+1}^M$).

Two things can be noted from these budget constraints. First: M_t is the money held by the public at the *beginning* of period t; hence, it is the amount of money available to be spent on bonds and capital in the AM. However, the amount of money held by the public during the PM, i.e., the amount of money available to be spent on goods, is $M_t + s_t^B (B_{t+1}^M - B_t^M)$. And the amount of money held by the public at the *end* of period t, i.e., at the end of the CM, equals M_{t+1}. Second, we can add up the budget constraints of the two authorities. If we also assume that $B_t^M = 0$, i.e., the monetary authority does not carry a balance sheet but simply makes seigniorage transfers to the fiscal authority in the CM, then the budget constraint of the consolidated government is exactly the one from Section 2.4.

As in the main body of the paper, let us proceed by ignoring cyclical concerns, and look at steady states. Assume that the fiscal authority is committed to increasing the supply of nominal bonds at (gross) rate μ^B, for a long period of time.[2] Does that mean that the long-run inflation rate will be μ^B? Not necessarily, because it is still the monetary authority that controls the money stock. But it is now impossible for monetary policy to achieve every point on the 'cone of policy options' derived in Section 3.3 and illustrated in Figure 5. Instead, the monetary authority is left with three options:

(i) Grow the money stock at (gross) rate $\mu^M < \mu^B$ (or shrink it if $\mu^M < 1$). In that case, B/M grows large, and eventually equilibrium must be in Region (A), where the policy rate is maximal, and governed by the rate of money growth: $j = i = \mu^M/\beta - 1$.

(ii) Grow the money stock at exactly $\mu^M = \mu^B$. In that case, any policy interest rate $j \in [0, i]$ is achievable for the monetary authority.

(iii) Grow the money stock at a faster rate than the supply of bonds: $\mu^M > \mu^B$. In that case, $B/M \to 0$, and eventually equilibrium must be in Region (C), where the policy interest rate is at the zero lower bound: $j = 0$.

This menu of monetary policy options is illustrated in Panel [a] of Figure W.2. In every case $i = \mu^M/\beta - 1$; that is, the monetary authority controls inflation. However, a benevolent monetary authority seeking to maximize social welfare will have strong incentives to match the money growth rate to the bond supply growth rate, because that is the only way the policy rate can be set to the first-best level. Unless, of course, the monetary authority can implement the Friedman rule; but since this requires $S < 0$, negative seigniorage, they may not be able to do this without cooperation from the fiscal authority.

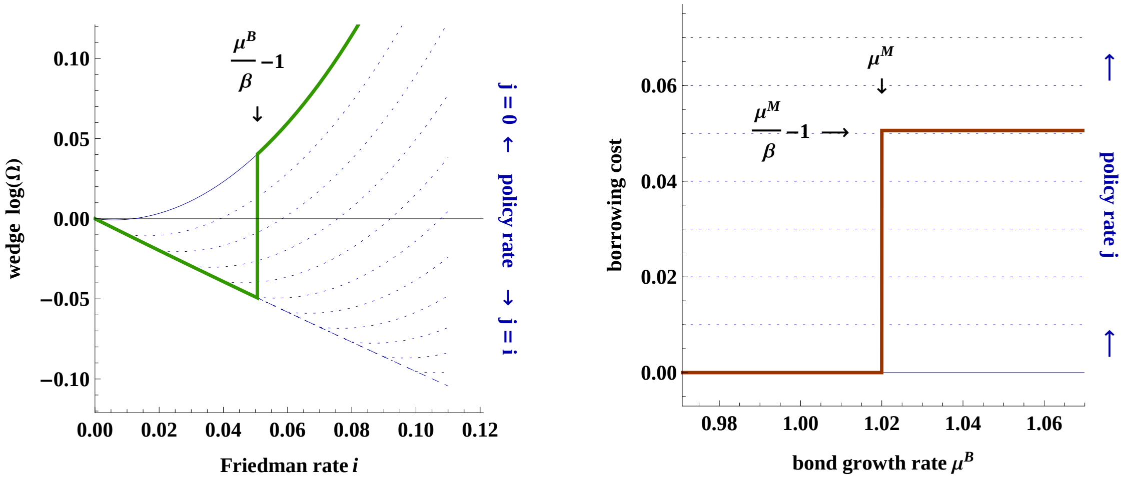
On the other hand, the fiscal authority may have incentives, too. Even if it is not fully benevolent, it probably still prefers low borrowing costs (policy interest rate j) to high ones. Panel [b] of Figure W.2 illustrates this. Taking the money growth rate μ^M as given, the fiscal authority is left with three options:

(i) Grow the bond supply at (gross) rate $\mu^B > \mu^M$, at least for a while (it cannot be forever due to the no-Ponzi condition). In that case, B/M grows large, and eventually equilibrium must be in Region (A), where the borrowing cost is maximal: $j = i = \mu^M/\beta - 1$.

(ii) Grow the bond supply at exactly $\mu^B = \mu^M$. In that case, the monetary authority chooses both i and j, and it is likely to choose $j < i$.

(iii) Grow the bond supply at a slower rate than the money stock: $\mu^B < \mu^M$. In that case, $B/M \to 0$, and eventually equilibrium must be in Region (C), where the borrowing cost is minimal: $j = 0$.

[2] Not, strictly speaking, forever; the fiscal no-Ponzi condition would be violated.



[a] Monetary policy options (green), constrained by bond growth rate μ^B

[b] Fiscal policy options (red), constrained by money growth rate μ^M

Figure W.2: Menu of long-run options for the monetary and fiscal authorities.

It is beyond the scope of this note to take a stand on the particular incentives that the two authorities may have, and to analyze this game exhaustively. But we learn a few simple lessons already. First, if the game is non-cooperative, clearly its outcome will hinge on which one of the two authorities has (or is perceived to have) greater commitment power. It stands to reason that the fiscal authority prefers low borrowing costs over high ones, hence it has a strong incentive to grow the bond supply in the long run at approximately the rate of inflation that the monetary authority prefers. However, if the fiscal authority is able to commit to a high rate of bond issuance, then a benevolent monetary authority also has an incentive to give in and accept the long-run inflation rate that the fiscal authority prefers.

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