# Why a Pandemic Recession *Boosts* Asset Prices<sup>\*</sup>

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### Lucas Herrenbrueck (Simon Fraser University)<sup>†</sup>

**Abstract**: Economic recessions are traditionally associated with asset price declines, and recoveries with asset price booms. Standard asset pricing models make sense of this: during a recession, dividends are low and the marginal value of income is high, causing low asset prices. Here, I develop a simple model which shows that this is not true during a recession caused by *consumption restrictions*, such as those seen during the 2020 pandemic: the restrictions drive the marginal value of income down, and thereby drive asset prices up, to an extent that tends to overwhelm the effect of low dividends. This result holds even if investors misperceive the economic forces at work.

JEL codes: E21, G12, I19

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Economists and market participants alike have been puzzled by how quickly, and how completely, stock prices have rebounded from their crash in the early days of the Covid-19 pandemic. Traditionally, we think of recessions as causing asset price declines, and recoveries causing booms, but by August 2020 stock indices in most countries had recovered beyond their previous peak, even as a conventional wisdom formed that the economic damage inflicted by the pandemic will be deep and long-lasting [10].

What can explain this disconnect? Contemporary financial analysts proposed three explanations [7, 12]: (a) asset markets are forward-looking, so high prices could just reflect investors' expectations of a quick end to the pandemic; (b) the kinds of big companies that are represented in the major stock indices are shielded from pandemic effects, or even stand to profit from them (e.g., Big Tech and Big Pharma); (c) asset values are being supported by central bank intervention. However, (a) is unlikely: data on dividend futures showed that as late as July 2020, dividend growth expectations remained depressed even as stock prices were already surging [3], so it appears that investors did not dispute the pessimistic economic forecasts. Arguments (b) and (c) are plausible, but do not explain why the asset market recovery has been so broad-based; by August 2020, the Russell 3000 index (which

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<sup>&</sup>lt;sup>†</sup> Contact: herrenbrueck@sfu.ca or @LHerrenbrueck. I would like to thank Zachary Bethune, Christina Chan, William Diamond, Athanasios Geromichalos, Francois Gourio, Gabriel Mihalache, Shiba Suzuki, Semih sl, and Shengxing Zhang for their valuable feedback, and Marieh Azizirad for excellent research assistance (funded by the Social Sciences and Humanities Research Council of Canada).



Figure 1: The "great disconnect": asset prices have boomed even though the global pandemic is ongoing. Data source: Yahoo Finance.

covers almost all of the US equity market) had passed its pre-pandemic peak, while bond markets and housing markets were setting records for high prices and low yields [4, 8].

In this paper, I construct a simple neoclassical asset pricing model in the spirit of the Lucas "tree" model [6], and I model various restrictions plausibly caused by a pandemic shock. The model suggests an alternative explanation for high asset prices: they are *caused* by the pandemic, not hindered by it, and more specifically they are caused by restrictions on consumption due to social distancing. (Whether distancing is voluntary or due to government mandates, as discussed by [2], is not relevant to the mechanism here.) In a typical recession, incomes fall, and households respond to shrinking budgets by reducing their consumption expenditure. This results in a rising marginal value of income, a falling desire to save, and a low valuation of financial assets. In a pandemic, on the other hand, households reduce consumption of socially-exposed goods and services in order to protect their health. Thus, it is the consumption restrictions that cause income reductions, and the result is a *falling* marginal value of income, an increasing desire to save (since additional income cannot be consumed, at least not in the way we want to most badly), and a high valuation of financial assets.<sup>1</sup>

Crucially, this is not a story of 'excess savings' but a story of increased 'demand for saving', which manifests even though incomes fall during the pandemic. In the simplest variant of the model, the aggregate supply of assets is fixed so that a pandemic-induced fall in consumption drags income along with it. In this case, a stronger 'desire to save' can be priced – people value savings more, thus asset prices surge – even though it cannot be satisfied in the aggregate, which is to say, on average the people in this economy are unable to increase

<sup>&</sup>lt;sup>1</sup> In its emphasis on a demand-side constraint which causes asset prices and dividends to move in opposite directions, the model is also related to [1]. In its emphasis on increased demand for saving dominating the effect of lower dividends, it is also related to [9].



Figure 2: In contrast to previous recessions (the most recent one of which is shown in the left figure), and defying the consumption smoothing motive predicted by macroeconomic theory, the Covid-19 recession (shown in the right figure) featured a larger percentage drop in consumption than in output. Data source: FRED.

how much they actually save.

Certainly, the reduced income causes lower asset dividends as well. However, unless the pandemic is expected to last for decades, the model shows that the effect of an increased desire to save easily dominates the effect of lower dividends. If there are restrictions on production in addition to consumption, the results are weakened and may get reversed, but only if both (a) production restrictions are tighter than consumption restrictions, and (b) the supply side of the economy is highly elastic in the short run. If agents misperceive the model, they will initially underprice assets as the pandemic hits, but the increased desire to save will eventually result in high assets prices even if nobody (within the model) understands the reason for this.

The paper is organized as follows. Section 1 develops the basic model and derives the principal results. Section 2 develops model variants with investment, multiple sectors, supply shocks, and beliefs. Section 3 discusses the limitations of the models and concludes.

## 1 The basic model

There are two states of the world:  $s_t \in \{0, 1\}$ . We call  $s_t = 1$  the "sick" or "pandemic" state and  $s_t = 0$  the "normal" state. There is a large measure of households and firms who take market prices as given. There is a single consumption good, which is produced using labor and a capital asset *in fixed supply*. The production function is:

$$y_t = k_t^{\alpha} h_t^{1-c}$$

The aggregate supply of capital is normalized to K = 1, but individual agents can buy and sell units of k at price  $q_t$ . In a period, capital yields a rental rate  $r_t$  and labor yields a wage rate  $w_t$ . Agents thus choose consumption  $c_t$  and labor supply  $h_t$  subject to the budget constraint:

$$c_t + q_t k_{t+1} = (r_t + q_t)k_t + w_t h_t$$

They seek to maximize the following standard utility function:

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \log c_t - \frac{1}{1+\eta} h_t^{1+\eta} \right)$$

However, the twist is that consumption in a pandemic state must also satisfy the constraint  $c_t \leq \hat{c}$ . This can be interpreted either as a physical or legal constraint (certain activities, like going to bars or traveling internationally, are prohibited), or a part of agents' preferences whereby consuming  $c_t > \hat{c}_t$  yields infinitely negative utility (people voluntarily avoid bars and air travel because of the infection risk). Either way, I assume that the constraint is slack in the normal state  $s_t = 0$ , and  $\hat{c}$  is so low that the constraint binds in the sick state  $s_t = 1$ .

Household decisions thus satisfy the following Bellman equation, in Lagrangian form:

$$V(k_t, s_t) = \max_{c_t, h_t, k_{t+1}} \left\{ \log(c_t) - \frac{1}{1+\eta} h_t^{1+\eta} + \beta \mathbb{E}_t \left\{ V(k_{t+1}, s_{t+1}) \right\} + \lambda_t [(r_t + q_t)k_t + w_t h_t - c_t - q_t k_{t+1}] + \mu_t s_t [\hat{c} - c_t] \right\}$$

where  $\lambda_t$  and  $\mu_t$  are the Lagrange multipliers on the budget and health constraints, respectively, which must satisfy the following sign and complementary slackness conditions:

$$\lambda_t \ge 0, \quad \mu_t \ge 0, \quad \mu_t s_t [\hat{c} - c_t] = 0, \text{ and } \lambda_t [(r_t + q_t)k_t + w_t h_t - c_t - q_t k_{t+1}] = 0.$$

The multiplier  $\lambda_t$  can thus be interpreted as the marginal value of income. (It turns out to be convenient to keep it in the equations, rather than immediately substituting it as we normally would in a growth model.)

**Proposition 1.** The solution to the household's problem satisfies  $c_t = \min\{1/\lambda_t, \hat{c}/s_t\}$ ,  $h_t^{\eta} = \lambda_t w_t$ , and the Euler equation:

$$\lambda_t q_t = \beta \mathbb{E}_t \Big\{ \lambda_{t+1} (r_{t+1} + q_{t+1}) \Big\}$$
(1)

*Proof: see the Appendix.* 

In a normal state, we have consumption equal to the inverse marginal value of income, but in a pandemic state, consumption is constrained by  $\hat{c}$ . The supply of labor is a positive



Figure 3: The "normal" state of the economy.

*Notes*: The "demand side" curve represents the first-order condition for consumption together with goods market clearing:  $y_t = c_t = 1/\lambda_t$ . The "supply side" curve represents the first-order conditions for labor supply and labor demand together with the production function:  $y_t = h_t^{1-\alpha} = [(1-\alpha)\lambda_t]^{(1-\alpha)/(\eta+\alpha)}$ . The "short-term asset pricing" curve represents the Euler Equation with expectations about the future held fixed:  $\lambda_t q_t = \text{constant}$ .

function of the marginal value of income and the wage, and the Euler equation for capital shows that when expectations about the future are held fixed, the asset price  $q_t$  depends negatively on the marginal value of income. The rest of the equilibrium conditions are standard: the rental rate on capital is  $r_t = \alpha h^{1-\alpha}$ , the aggregate labor demand curve is  $w_t = (1-\alpha)h_t^{-\alpha}$ , and market clearing in the goods market requires  $c_t = y_t = h_t^{1-\alpha}$  (since the capital stock is fixed at 1). An **equilibrium** is defined to be any bounded sequence of  $\{c_t, h_t, r_t, w_t, \lambda_t, q_t\}_{t=0}^{\infty}$  satisfying these equations, for a given belief about states  $\{s_t\}_{t=0}^{\infty}$ .

#### Never pandemic

In a normal state, the optimality condition  $c_t = 1/\lambda_t$  combined with the market clearing equations yields the following equilibrium:

$$c_t = \bar{c} \equiv (1 - \alpha)^{\frac{1 - \alpha}{1 + \eta}} \qquad r_t = \bar{r} \equiv \alpha \bar{c}$$

$$h_t = \bar{h} \equiv (1 - \alpha)^{\frac{1}{1 + \eta}} \qquad \lambda_t = \bar{\lambda} \equiv \frac{1}{\bar{c}}$$
(2)

To derive a simple benchmark for the price of capital, suppose that the normal state is believed to last forever with no risk of a future pandemic. Plugging the solutions back into Equation (1), and using  $q_t = q_{t+1}$ , we obtain the never-pandemic price of capital:

$$\bar{q} = \frac{\alpha\beta}{1-\beta}\bar{c} \tag{3}$$

### **Forever pandemic**

However, a pandemic *did* strike in 2020. Solving the optimality and market clearing conditions together with  $c_t = \hat{c}$ , in a pandemic state we have:

$$c_t = \hat{c} \qquad r_t = \alpha \hat{c}$$

$$h_t = (\hat{c})^{\frac{1}{1-\alpha}} \qquad \lambda_t = \frac{1}{1-\alpha} (\hat{c})^{\frac{\eta+\alpha}{1-\alpha}}$$
(4)

Thus, during the ongoing pandemic all real variables are characterized by the constraint  $\hat{c}$  alone. The only variable that requires knowing more than that is the price of capital, because that depends on whether agents believe the pandemic will persist or end soon.

If the pandemic is expected to persist forever, then of course we have a steady state with  $q_t = q_{t+1}$ . Plug this into Equation (1), evaluate at pandemic values, and we obtain the forever-pandemic asset price value which we can call  $\hat{q}$ :

$$\hat{q} = \frac{\alpha\beta}{1-\beta}\hat{c} \tag{5}$$

### Pandemic is expected to last for *n* periods

With these tools, we can analyze what would happen if the pandemic was believed to last for  $n \ge 0$  more periods (excluding the current period). Define  $q^n$  to be the price of the asset in this case:

**Proposition 2.** Expressed in relation to the long-term normal-state asset price  $\bar{q}$  (Equation 3) and consumption  $\bar{c}$  (Equation 2), the pandemic-era asset price  $q^n$  satisfies:

$$\frac{q^n}{\bar{q}} = (1 - \beta^n)\frac{\hat{c}}{\bar{c}} + \beta^n \left(\frac{\hat{c}}{\bar{c}}\right)^{-\frac{\eta + \alpha}{1 - \alpha}} \tag{6}$$

Proof: see the Appendix.

Equation (6) is the main result of this paper. Notice what happens if the pandemic lasts forever  $(n \to \infty)$ : consumption is depressed forever, hence economic activity is depressed forever, and so is the price of capital. But if the pandemic is expected to be short-lasting (n = 0 in particular), the depressed economic activity results in a boost to asset prices, since the exponent on the last term is negative. For realistic values of the discount factor  $\beta$  and the share of quickly-adjustable factors of production  $(1 - \alpha)$ , the (negative) contribution



Figure 4: The "pandemic" state of the economy.

*Notes*: The "demand side" curve represents the first-order condition for consumption together with goods market clearing:  $y_t = c_t = \min\{\hat{c}, 1/\lambda_t\}$ . The "supply side" curve represents the first-order conditions for labor supply and labor demand together with the production function:  $y_t = h_t^{1-\alpha} = [(1-\alpha)\lambda_t]^{(1-\alpha)/(\eta+\alpha)}$ . The "short-term asset pricing" curve represents the Euler Equation with expectations about the future held fixed ( $\lambda_t q_t = \text{constant}$ ), whereas the "long-term asset pricing" curve represents the Euler equation solved in steady state (5).

of the first term is dominated by the (positive) contribution of the second term – even if the pandemic was expected to last, say, three or four years.

What is the intuition for this striking result? The key lies in the level of  $\lambda$ , the marginal value of income or wealth, during the pandemic. Generally in macroeconomics, this value is inversely related to consumption – such as here,  $\overline{\lambda} = 1/\overline{c}$  in the normal state – a result so basic that it has become part of the 'deep wiring' of a macroeconomist's thinking engine. However, **in a pandemic, consumption is not constrained by wealth, but by health**. Thus, the only value of an increase in wealth is that it helps the household work less, and avoid the disutility of working. But this disutility falls when the economy is constrained – indeed, Equation (4) confirms that  $\lambda$  is positively related to the consumption constraint.

Now, standard theory suggests that asset prices are determined by two things: expected dividends, and the value of deferring wealth into the future. It is true that dividends ( $r_t = \alpha c_t$  in the simple model, perfectly correlated with aggregate consumption) are lower in the pandemic ( $\hat{c}$  vs  $\bar{c}$ ), and this channel becomes more important the longer the pandemic is expected to last. However, if the pandemic is not expected to last beyond a few years, what is much more important than dividends is the motivation to defer spending until normal activity can resume. Figure 5 illustrates this result with a numerical example.



Figure 5: Asset prices during a pandemic; unless the pandemic is expected to last decades, the effect of lower dividends is dominated by the effect of lower marginal value of income.

*Notes*: We assume the pandemic hits as a surprise in period 0, causes consumption to be restricted to 90% of its normal level, and is immediately understood to last through period 1 and end in period 2. Parameters:  $\alpha = 1/3, \eta = 1, \beta = 0.95$ .

#### Pandemic is expected to end at random date

We can do a similar analysis under the simplifying assumptions that we start in the pandemic, each period the pandemic ends with probability  $1 - \pi$  and persists otherwise, and once the pandemic is over it never returns. We denote the asset price in this scenario by  $q^{\pi}$ (note that  $\pi$  is a label here, not an exponent). After some algebra, we obtain:

$$\frac{q^{\pi}}{\bar{q}} = \frac{(1-\beta)\pi}{1-\beta\pi} \cdot \frac{\hat{c}}{\bar{c}} + \frac{1-\pi}{1-\beta\pi} \left(\frac{\hat{c}}{\bar{c}}\right)^{-\frac{\eta+\alpha}{1-\alpha}}$$

Again, unless  $\pi$  is of similar magnitude to  $\beta$  (meaning the pandemic is expected to persist for decades), the second term dominates and the pandemic causes a boost in asset prices.

## 2 Bells and whistles

Certainly, the basic model from Section 1 is just that, basic. In this section I solve four extensions – investment, multiple sectors, capital obsolescence, and incorrect beliefs by agents within the model – and discuss how they modify the conclusions of the basic model.

#### 2.1 The supply of assets is not fixed

In the basic model, the supply of capital goods is fixed and there is no investment. This assumption serves to throw into sharp relief the result that high asset prices are not necessarily a consequence of *increased saving* during the pandemic, only of an *increased desire to save*. In reality, of course, storing resources for the future via investment is possible, so one might wonder if the results of this paper survive such an extension.

In order to construct a model with an imperfectly elastic capital stock that also admits closed-form solutions, I modify the basic model by assuming that there are two types of capital goods: a fixed type  $(k^F)$  that is in constant supply and lasts forever, as before, and an elastic type  $(k^E)$  that fully depreciates each period and must be restocked via investment.<sup>2</sup> The production function is:

$$y_t = (k_t^F)^{\alpha^F} (k_t^E)^{\alpha^E} h_t^{1-\alpha}$$

(where we define  $\alpha \equiv \alpha^F + \alpha^E$ ) and the resource constraints for the two types of capital are:

$$k_{t+1}^F = 1, \quad k_{t+1}^E = y_t - c_t$$

Because the aggregate supply of *F*-capital is fixed, its price  $q_t^F$  is determined by an Euler equation; on the other hand, since the rate of transformation of output goods into *E*-capital is constant at 1, so is the price of *E*-capital:  $q_t^E = 1$ . We define the market capitalization of the overall capital stock (which is the closest model counterpart to a stock market index like the S&P 500 or Russell 3000) to be:

$$Q_t \equiv q_t^F \cdot k_t^F + q_t^E \cdot k_t^E = q_t^F + k_t^E$$

In a period, the two types of capital yield rental rates  $r_t^F$  and  $r_t^E$ ; the rest is as before. The household's problem satisfies the following Bellman equation:

$$\begin{split} V(k_t^F, k_t^E, s_t) &= \max_{\substack{c_t, h_t, \\ k_{t+1}^F, k_{t+1}^E}} \left\{ \log(c_t) - \frac{1}{1+\eta} h_t^{1+\eta} + \beta \,\mathbb{E}_t \big\{ V(k_{t+1}^F, k_{t+1}^E, s_{t+1}) \big\} \\ &+ \lambda_t [(r_t^F + q_t^F) k_t^F + r_t^E k_t^E + w_t h_t - c_t - k_{t+1}^E - q_t^F k_{t+1}^F] + \mu_t s_t [\hat{c} - c_t] \Big\} \\ &\text{subject to: } \lambda_t \ge 0, \ \mu_t \ge 0, \ \mu_t s_t [\hat{c} - c_t] = 0, \\ &\text{and } \lambda_t \left[ (r_t^F + q_t^F) k_t^F + r_t^E k_t^E + w_t h_t - c_t - k_{t+1}^E - q_t^F k_{t+1}^F \right] = 0. \end{split}$$

The solution to the household's problem and definition of equilibrium proceed analogously to Section 1, so I omit the details here and go straight to the equilibrium equations. The Euler equations for the two types of capital are:

$$\lambda_t q_t^F = \beta \mathbb{E}_t \Big\{ \lambda_{t+1} (r_{t+1}^F + q_{t+1}^F) \Big\} \quad \text{and} \quad \lambda_t = \beta \mathbb{E}_t \Big\{ \lambda_{t+1} r_{t+1}^E \Big\};$$

<sup>&</sup>lt;sup>2</sup> In standard neoclassical growth models, investment is perfectly elastic and the price of capital goods is always constant. Such a model is therefore a non-starter for a paper studying the evolution of asset prices. The typical alternative is a Tobin's-Q model where investment is subject to frictions (e.g., [5]), but such models do not allow pencil-and-paper solutions even for simple shocks like the one studied here.

recall that the "elastic" type of capital fully depreciates each period, while the "fixed" type of capital never depreciates at all. In equilibrium, the rental rates on the two types of capital satisfy:  $r_t^F = \alpha^F y_t$  and  $r_t^E = \alpha^E y_t/k_t^E$ . Substituting these, the Euler equations become:

$$\lambda_t q_t^F = \beta \mathbb{E}_t \Big\{ \lambda_{t+1} (\alpha^F y_{t+1} + q_{t+1}^F) \Big\}$$
(7)

$$\lambda_t = \beta \mathbb{E}_t \Big\{ \alpha^E \lambda_{t+1} y_{t+1} / k_{t+1}^E \Big\} \quad \leftrightarrow \quad \lambda_t k_{t+1}^E = \beta \alpha^E \mathbb{E}_t \Big\{ \lambda_{t+1} y_{t+1} \Big\}$$
(8)

Since  $k_{t+1}^E = y_t - c_t$  is simply the amount of resources invested in period t, it is pre-determined at t+1 and can be pulled outside the expectations operator.

As in the basic model, we use labor demand, labor supply, and the production function to eliminate  $w_t$  and  $h_t$ , yielding an equation relating output to the marginal value of income:

$$y_t = \left(k_t^E\right)^{\frac{\alpha^E(\eta+1)}{\eta+\alpha}} \cdot \left[(1-\alpha)\lambda_t\right]^{\frac{1-\alpha}{\eta+\alpha}}$$
(9)

Thus, output in period t depends on investment in the past  $(k_t^E)$  and on the marginal value of income in the present  $(\lambda_t)$ . However, note that  $y_t$  is the only equilibrium variable that *directly* depends on the past; every other variable depends only on expectations of the future, thus we can solve the rest of the equilibrium in closed form.

In a normal state,  $\lambda_t = 1/c_t$ . We guess that households consume a fixed fraction of their income, and invest the rest:  $c_t = (1 - \phi)y_t$  and  $k_{t+1}^E = \phi y_t$ . Thus,  $\lambda_t y_t = 1/(1 - \phi)$ , which we substitute into the Euler equations (7)-(8) to obtain:

$$\frac{q_t^F}{y_t} = \beta \mathbb{E}_t \left\{ \alpha^F + \frac{q_{t+1}^F}{y_{t+1}} \right\} \quad \Rightarrow \quad q_t^F = \beta \alpha^F / (1 - \beta) \cdot y_t$$
$$\frac{\phi}{1 - \phi} = \beta \mathbb{E}_t \left\{ \frac{\alpha^E}{1 - \phi} \right\} \quad \Rightarrow \quad \phi = \beta \alpha^E$$

The solution in the first line is obtained by iterating the Euler equation forward, and it says simply that the market price of fixed capital is proportional to output. The solution in the second line verifies the guess of a constant investment rate  $\phi$ .

During the pandemic, the investment rate is clearly not constant, since consumption is exogenously determined by the bound  $\hat{c}$ . To simplify the analysis, assume for this subsection only that the pandemic is known to last for one period, i.e., it begins and ends in period 0 and never returns. What happens to investment,  $k_1^E$ ? Via Equation (8):

$$k_1^E = \frac{\beta \alpha^E}{\lambda_0} \mathbb{E}_0 \left\{ \lambda_1 y_1 \right\} = \frac{\beta \alpha^E}{1 - \beta \alpha^E} \frac{1}{\lambda_0}$$
(10)

because the economy is back in its normal state from period 1 on. Thus, investment in the

pandemic is a decreasing function of the marginal value of income. Now, consumption equals output minus investment:

$$c_0 = y_0(\lambda_0) - k_1^E(\lambda_0)$$
(11)

From Equation (9), we know that  $y_0$  is an increasing function of  $\lambda_0$ , and we have just shown that  $k_1^E$  is a decreasing function of  $\lambda_0$ . Thus, consumption during the pandemic ( $c_0$ ) must be in a *positive* relationship with the marginal value of income ( $\lambda_0$ ), just as it was in the basic model. Since the pandemic reduces consumption down to the level  $\hat{c} < \bar{c}$ , this implies that the key result  $\lambda_0 < \bar{\lambda}$  is preserved (even though Equation (11) can no longer be solved in closed form, except in the very specific case of  $\eta = 1 - 2\alpha$ ).

Thus, Equation (9) confirms that even in the extended model, output still falls during the pandemic, though it now falls by less than consumption because (via Equation 10) investment in the elastic type of capital increases.

Finally, what happens to the variable we are most interested in: the market capitalization of the capital stock in this economy,  $Q_t = q_t^F + k_t^E$ ? In period 0,  $k_t^E$  is pre-determined (say, at the no-pandemic steady-state level  $\beta \alpha^E \bar{y}$ ). For  $q_0^F$ , we start with the Euler equation (7) and substitute our previous results:

$$\begin{split} \lambda_0 q_0^F &= \beta \mathbb{E}_0 \Big\{ \lambda_1 (\alpha^F y_1 + q_1^F) \Big\} = \frac{\beta}{1 - \beta \alpha^E} \mathbb{E}_0 \Big\{ \alpha^F + \frac{q_1^F}{y_1} \Big\} = \frac{\beta}{1 - \beta \alpha^E} \left( \alpha^F + \frac{\beta \alpha^F}{1 - \beta} \right) \\ &= \frac{\beta \alpha^F}{(1 - \beta)(1 - \beta \alpha^E)} \quad \Rightarrow \quad \boxed{q_0^F = \frac{\text{constant}}{\lambda_0}} \end{split}$$

Since  $\lambda_0 < \overline{\lambda}$ , the result from the basic model that asset prices increase during the pandemic is confirmed even when the asset supply is elastic.

What happens after the pandemic is over? In the basic model, the economy returns to steady state immediately, but here there is some persistence. We know that during the pandemic, investment rises *above* the previous steady state. Thus, output after the pandemic is also above the normal steady state (thanks to the rise in investment) and smoothly converges back towards it. Since we have shown that during the normal state,  $q_t^F$  is proportional to output  $y_t$ , and  $k_t^E$  is proportional to  $y_{t-1}$  (if period t - 1 was also normal), total asset market capitalization  $Q_t$  behaves like a weighted moving average of current and past income  $y_t$  and  $y_{t-1}$ , except for the pandemic period itself; thus, in contrast to the basic model, the extended model predicts that the asset market boom may not come to a sudden end once the pandemic is over, but will taper off more smoothly.

#### 2.2 Multiple sectors

Of course, the effect of the 2020 pandemic has not been to force a reduction in all kinds of consumption spending equally. Some spending on affected goods (air travel and movie theaters) has been diverted to others (hand sanitizer and yoga mats). To capture this, I augment the model from Section 1 with two sectors producing different goods: "social" consumption  $c_t^S$  which is subject to the health constraint  $c_t^S \leq \hat{c}/s_t$ , and "private" consumption  $c_t^P$  which is not constrained in this way. The utility function is changed to:

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \sigma \log c_t^S + (1-\sigma) \log c_t^P - \frac{1}{1+\eta} h_t^{1+\eta} \right)$$

so that, ideally, households want to spend a fraction  $\sigma \in (0, 1)$  of their income on social goods and the remainder on private goods. The resource constraint is:

$$c_t^S + c_t^P = y_t = k_t^\alpha h_t^{1-\alpha}$$

Since this is a simple extension of the basic model, I skip the Bellman equation and go straight to the solution. It turns out that output must satisfy the equation:

$$\hat{c} + (1 - \sigma)(1 - \alpha)y_t^{-\frac{\eta + \alpha}{1 - \alpha}} = y_t$$

This equation clearly has a unique solution for  $y_t$ , but it cannot be solved in closed form except in a few special cases. One such special case is  $\eta \to \infty$ , meaning that labor supply and output are both fixed at 1, and the only problem in this economy is to allocate consumption between social and private consumption.<sup>3</sup> In that case, the solution for the asset price during a pandemic (which is again expected to last for  $n \ge 0$  more periods) would be:

$$\frac{q^n}{\bar{q}} = (1 - \beta^n) \cdot 1 + \beta^n \cdot \frac{1 - \hat{c}}{1 - \sigma}$$

This time, there is only a positive effect (through  $\hat{\lambda} \downarrow$ ); because output is fixed, so are dividends. And as one would expect, the effect of a restriction on social consumption is strongest when the social sector is a big share ( $\sigma$ ) of the economy; specifically, the elasticity of the short-pandemic stock price  $q^0$  with respect to  $\hat{c}$ , evaluated near the no-restrictions steady state  $\bar{c}^S = \sigma$ , is:

$$\frac{d\log(q_0)}{d\log(\hat{c})}\Big|_{\hat{c}\to\sigma} = -\frac{\sigma}{1-\sigma}$$

<sup>&</sup>lt;sup>3</sup> Another special case is  $\eta = 1 - 2\alpha$ , where the exponent  $(\eta + \alpha)/(1 - \alpha)$  equals 1 so the model can be solved in closed form as well. In that case, the model makes the intuitive prediction that the impact of the pandemic is mixed between *S*-consumption falling, output as well but less so, and *P*-consumption increasing.

The lesson of this equation is that when the supply side of the economy is inelastic, the impact of pandemic restrictions on asset prices can be arbitrarily large, even when the intensive margin of restrictions ( $\hat{c} < \bar{c}^S$ ) is small; what matters is the extensive margin, or *how many* kinds of consumption are restricted, moreso than by *how much*.

Summing up, this extension shows that main result survives even when some sectors are not affected by the pandemic: the forced reallocation of consumption from "social" to "private" goods also causes the marginal value of income to decrease. Intuitively, this makes sense: after all, an extra unit of spending could only go towards something we already have enough of (how many webcams or cuisinarts does one need?) and not on the things we most badly crave (travel and socializing).

### 2.3 Capital obsolescence

The model in Section 1 is simple and based on very standard macroeconomic principles. So, why does our intuition seem to dictate that the pandemic should decrease stock prices? One of the reasons might be the fact that social distancing restrictions – whether voluntary or not – do not only affect the availability of consumption goods, but also the usefulness of certain kinds of capital. For example, restaurants are forced to operate at reduced capacity, and sports arenas and convention centers are kept empty. Grocery stores and airports are open, but their interiors have been retrofitted at high costs.

Here, we can capture this channel by assuming that during the pandemic state  $s_t = 1$ , only a fraction of the capital stock,  $\kappa < 1$ , can be used. That is to say, an agent holding  $k_t$  units of capital during the pandemic is only able to rent out (and collect returns on)  $\kappa k_t$  of them. The aggregate stock of capital remains normalized at K = 1 throughout this exercise.

#### 2.3.1 No consumption restrictions

To begin with, we assume that there are no consumption restrictions. Then, consumption equals the inverse marginal value of income:  $c_t = 1/\lambda_t$ . As before, the labor supply curve is  $h_t^{\eta} = \lambda_t w_t$ , and the aggregate labor demand curve is  $w_t = (1 - \alpha)y_t/h_t$ . The aggregate resource constraint is  $c_t = y_t = \kappa^{\alpha} h_t^{1-\alpha}$  during the pandemic, and  $c_t = y_t = h_t^{1-\alpha}$  outside of it. Either way, the rental rate on total capital is  $r_t = \alpha y_t$ ; during the pandemic, not all capital is usable (so we need to multiply by  $\kappa$ ), but the marginal value of *usable* capital is inversely proportional to the ratio of usable capital per unit of output (so we divide by  $\kappa$ ). Solving, we obtain the key equations:

$$y_t = c_t = \kappa^{\alpha} \cdot \bar{c} \qquad \Rightarrow \qquad r_t = \alpha \bar{c} \cdot \kappa^{\alpha}$$
$$\lambda_t = 1/c_t \qquad \Rightarrow \qquad \lambda_t = \alpha/r_t$$

We plug these results into the Euler equation (1), and notice that  $\lambda_{t+1}r_{t+1}$  simplifies to a constant  $\alpha$ . Thus,  $\lambda_t q_t = \beta \mathbb{E}_t \{ \alpha + \lambda_{t+1}q_{t+1} \}$ , and we can simply iterate on  $\lambda_t q_t$  to obtain:

$$\lambda_t q_t = \frac{\alpha \beta}{1 - \beta} \qquad \Rightarrow \qquad \frac{q_t}{\bar{q}} = \frac{c_t}{\bar{c}} = \kappa^{\alpha}$$
 (12)

Thus, current asset prices only depend on current consumption. In particular, this implies that during a pandemic where only a fraction  $\kappa$  of all capital can be used to earn returns, stock prices should be scaled down by a factor  $\kappa^{\alpha}$ . Furthermore, it implies that the duration of the pandemic is irrelevant; and, more than that, it is irrelevant whether the pandemic-induced loss of capital is believed to be temporary or permanent!

The reason for this strong result is of course our assumption of logarithmic utility; capital obsolescence causes both an income and a price effect, and these two effects offset exactly. If the obsolete capital is gone forever, then expected future returns fall but the marginal value of saving  $(\lambda_{t+1}/\lambda_t)$  is flat. If the capital is only temporarily disabled, then expected future returns are preserved but the marginal value of saving falls by an equal amount.

To be sure, this is a special case, and we could analyze variations with more general intertemporal preferences. Nevertheless, as long as these variations do not depart too far from the logarithmic benchmark, the results are clear: (i) if the only effect of the pandemic is that some fraction of the capital stock becomes unusable, capital prices fall; (ii) the loss of usable capital is passed through to stock prices with elasticity  $\alpha$ , the elasticity of output with respect to the relevant type of capital; (iii) it does not matter whether the disruption is temporary or permanent.

#### 2.3.2 Capital and consumption restrictions combined

Naturally, since the 2020 pandemic has caused restrictions to both consumption and productive capacity, we should investigate the combined effect of these restrictions. To do so, I assume that  $\hat{c} < \kappa^{\alpha} \bar{c}$ ; that is to say, consumption is restrained even below the level that can be achieved with the reduced capacity. In this case, during a pandemic state  $s_t = 1$ , output is again determined by consumption demand ( $y_t = c_t = \hat{c}$ ), and so are returns on total capital ( $r_t = \alpha y_t = \alpha \hat{c}$ ). After some algebra, the marginal value of income during the pandemic is:

$$\lambda_t \Big|_{s_t=1} = \frac{1}{1-\alpha} (\hat{c})^{\frac{\eta+\alpha}{1-\alpha}} (\kappa)^{-\frac{\alpha\eta+\alpha}{1-\alpha}},$$
(13)

which is, confirming the results from the previous models, increasing in the allowed fraction of consumption  $\hat{c}/\bar{c}$  and decreasing in the fraction of usable capital  $\kappa$ .

As before, suppose that the pandemic is known to persist for another  $n \ge 0$  periods after the current one; iterating Equation (1) just like we did for Proposition 2, we obtain the following ratio of stock prices to their long-term pre-pandemic value:



Figure 6: The "pandemic" state, with restrictions on consumption and capital use.

*Notes*: The "demand side" curve represents the first-order condition for consumption together with goods market clearing:  $y_t = c_t = \min\{\hat{c}, 1/\lambda_t\}$ . The "supply side" curve represents the first-order conditions for labor supply and labor demand together with the production function:  $y_t = \kappa^{\alpha} h_t^{1-\alpha} = \kappa^{\alpha} [(1-\alpha)\lambda_t]^{(1-\alpha)/(\eta+\alpha)}$ . The "short-term asset pricing" curve represents the Euler Equation with future expectations held fixed ( $\lambda_t q_t = \text{constant}$ ), whereas the "long-term asset pricing" curve represents the Euler equation solved in steady state (5).

$$\frac{q^n}{\bar{q}} = (1 - \beta^n)\frac{\hat{c}}{\bar{c}} + \beta^n \left(\frac{\hat{c}}{\bar{c}}\right)^{-\frac{\eta+\alpha}{1-\alpha}} (\kappa)^{\frac{\alpha\eta+\alpha}{1-\alpha}}$$
(14)

Comparing this equation with the earlier result (6), the effect of the pandemic on the dividend component of the equation is exactly the same; as long as the consumption restriction is the binding constraint, dividends are proportional to aggregate consumption, no matter what happens to capital. But as explained earlier, given reasonable values for  $\beta$  and unless the pandemic is expected to persist for many years, what matters for stock prices *during* the pandemic is the final value  $q_0/\bar{q}$ , the *last price before exit* from the pandemic. Here, the two restrictions push in opposite directions: the restriction on consumption ( $\hat{c} < \bar{c}$ ) lowers the marginal value of income, while the restriction on capital use ( $\kappa < 1$ ) increases it.

In a pandemic like the one in 2020 where both restrictions operate, which one wins? In principle, this is of course a quantitative question, but even just with theory we can say a bit by comparing the elasticities. It turns out that the elasticity on  $\hat{c}$  (in absolute value) exceeds the elasticity on  $\kappa$  by exactly  $\eta$ , the inverse elasticity of the labor supply. This means that when the labor supply is elastic (so that  $\eta \rightarrow 0$ ), consumption and capital restrictions are about equally strong in the magnitude of their effect on stock prices. When the labor supply is inelastic, on the other hand ( $\eta \rightarrow \infty$ ), then the effect of consumption restrictions will

dominate and even a small consumption restriction can drive asset prices arbitrarily high.

For concreteness, consider a few examples (and for simplicity, assume that the pandemic lasts for only one period in each case):

- (E1) The pandemic reduces both consumption and the usable capital stock by one percent  $(\hat{c}/\bar{c} = \kappa = 0.99)$ . In this case, since  $\alpha < 1$ , we have  $\hat{c} < \kappa^{\alpha}\bar{c}$  and thus the constraint on consumption binds. The stock price  $q^0$  increases by  $\eta$  percent above  $\bar{q}$ .
- (E2) The usable capital stock falls by ten percent ( $\kappa = 0.9$ ), but people can satisfy the health constraint if they reduce consumption by three percent ( $\hat{c}/\bar{c} = 0.97$ ). If  $\alpha \ge 0.3$ , then consumption falls by  $10\alpha$  percent which is *more* than three percent, so the health constraint does not bind. Stock prices fall by  $10\alpha$  percent, the same as consumption.
- (E3) The usable capital stock falls by four percent ( $\kappa = 0.96$ ), and in order to stay healthy people must reduce consumption by two percent ( $\hat{c}/\bar{c} = 0.98$ ). Also, suppose  $\alpha = 1/3$  and  $\eta = 1$ . Then, stock prices stay exactly the same compared to both before and after the pandemic.
- (E4) The usable capital stock falls by three percent ( $\kappa = 0.97$ ), but in order to stay healthy people must reduce consumption by nine percent ( $\hat{c}/\bar{c} = 0.91$ ). Also, suppose  $\alpha = 1/3$  and  $\eta = 1$ . Then, stock prices increase by  $[(9 3\alpha)\eta + 6\alpha]/(1 \alpha) = 15$  percent.

These examples illustrate that when the consumption restriction  $\hat{c}/\bar{c}$  and the capital usability restriction  $\kappa$  are similar in magnitude, the effect of the consumption restriction tends to win out and cause stock prices to go up during the pandemic. In order for stock prices to go down, we would need (a) the capital restriction to be much severe than the consumption restriction; (b) the capital elasticity in the production function,  $\alpha$ , to be large; (c) the elasticity of short-term labor supply,  $1/\eta$ , to be large as well.

## 2.4 Incorrect beliefs

A model first proposed in August of 2020 cannot claim that investors in March of 2020, when stock prices crashed, knew that new model. Instead, it is plausible that during the early days of the pandemic when it became clear that it would cause deep and persistent economic damage, investors used familiar models to predict its effect on asset returns and prices. We can never be sure what exactly investors were thinking, but media reports at the time indicate that the asset price rally in Spring 2020 came as a surprise to most [7, 12].

In order to capture this fact in a simple, reduced-form bounded rationality way, I assume for this section that (a) reality is described by the model from Section 2.3.2, but (b) agents believe that they are living in the world of Section 2.3.1 and are unaware of the ultimate effects of the health constraint  $c_t \leq \hat{c}$  on asset prices as the pandemic hits.

To keep things simple, I also assume that the pandemic hits (as a complete surprise) in period 0, at which point everyone believes with certainty that the pandemic will persist

through period 1 and be over in period 2. (The argument that follows will mainly focus on asset prices in periods 0 and 1, thus it does not matter much whether the pandemic is actually over in period 2.) Thus, the economy is expected to be in the normal steady state from period 2 on:

$$\lambda_t q_t = \bar{\lambda} \bar{q} \quad \forall t \ge 2$$

At time 0, agents believe that in period 1, the model from Section 2.3.1 will apply, hence the asset price in period 1 will be  $\tilde{q}_1 = \kappa^{\alpha} \bar{q}$ . Thus, in period 0, they evaluate the Euler equation:

$$\lambda_0 q_0 = \beta \mathbb{E}_t \{ \lambda_1 (r_1 + \tilde{q}_1) \}$$

They also believe that period 1 is still in the pandemic, as is period 0, thus they conclude that  $\lambda_0 = \lambda_1$  (whatever that value may turn out to be); hence,  $\lambda$  drops out of the Euler equation. Finally, in accordance with their model of the world, agents believe that  $r_1 = \kappa^{\alpha} \bar{r}$  because of the ongoing restrictions on the use of capital during the pandemic. In that case, their willingness to pay for capital in period 0 is  $q_0 = \kappa^{\alpha} \bar{q}$ , the same as they believe will be the price in period 1.

However, once period 1 comes around, agents' marginal value of saving will be low, not high, due to the ongoing consumption restrictions (see Equation 13). Their willingness to pay for capital will be:

$$q_1 = \frac{\lambda_2}{\lambda_1} \bar{q} = \left(\frac{\hat{c}}{\bar{c}}\right)^{-\frac{\eta+\alpha}{1-\alpha}} \left(\kappa\right)^{\frac{\alpha\eta+\alpha}{1-\alpha}} \cdot \bar{q},$$

the 'correct' asset price as per Equation (14). Thus, the trajectory of asset prices satisfies:

$$\bar{q} > q_0 < q_1 \stackrel{?}{\gtrless} \bar{q} \tag{15}$$

They go through a zigzag pattern, falling at the onset of the pandemic, rising (possibly above the steady state) near its end, and returning to the old steady state once the pandemic is over. Figure 7 illustrates this result with a numerical example.

The point here is not that agents 'learn' in period 1 that they were wrong about the model of the pandemic. On the contrary, finding the 'correct' price in period 1 requires only that agents have correct beliefs about period 2 (the pandemic is over and the economy returns to steady state), and respond optimally to their own individual constraints (budget and health). They may observe the zigzag pattern for asset prices, but incorrectly attribute it to changing beliefs about the course of the pandemic.



Figure 7: Asset prices during a pandemic when agents have incorrect beliefs.

*Notes*: We assume the pandemic hits as a surprise in period 0, causes both consumption and capital use to be restricted to 80% of their normal levels, and is immediately understood to last through period 1 and end in period 2; agents do *not* understand the effect of consumption restrictions on asset prices, but otherwise act optimally. Parameters:  $\alpha = 1/3$ ,  $\eta = 1$ ,  $\beta = 0.95$ .

## 3 Discussion

In this paper, I develop a simple variation of the standard neoclassical growth model. In the benchmark version, there are only two changes: first, a pandemic shock forces everybody to *reduce consumption* below the steady-state value, and second, capital is in fixed supply. This second assumption makes capital similar to a "Lucas tree" [6]; however, in Lucas' model, trees are the only factor of production, whereas here it turns out to be important that there is also an elastic factor of production (otherwise, asset prices blow up to infinity). For the purposes of my model, I call that factor "labor", but it really represents any input into production of which the supply can be quickly adjusted.

The model implies that a pandemic causes a decrease in the marginal value of current income, which can be interpreted as an increased demand for saving, and which is translated into high asset prices. This result is not particularly dependent on how low asset dividends fall during the pandemic; it only requires that the pandemic be short (in the sense of not lasting more than a few years), and that consumption restrictions be at least as severe as restrictions on supply. It also has nothing to do with central bank intervention in asset markets; the point is that the pandemic increases the *demand for saving instruments in general*, which drives up their prices, so if a central bank swaps one kind of saving instrument (stocks and bonds) for another (money), this does little to satisfy the increased demand overall.

The model, simple as it is, does miss one big ingredient in real-world asset markets: leverage. If leverage is high, this could provide one reason for stock prices during the pandemic to stay low, or not rise as high as the model predicts. For example, a firm with a leverage ratio of 10 will see its dividends fall by 50 percent even if the aggregate economy only shrinks by 5 percent, and it is only the 5 percent that cause a higher demand for saving, not the 50. If the aggregate economy shrinks even more, the firm goes bankrupt so its asset value hits zero and never recovers, even after the pandemic. Thus, if we expect a pandemic recession to be so severe as to cause widespread *bankruptcies*, then the increased demand for saving instruments would not necessarily be enough to save the stock market from collapse.

### APPENDIX

*Proof of Proposition 1.* We define the set of possible histories to be  $\{\theta_t : \theta_t = \{s_0, \ldots, s_t\}, s_t \in \{\theta_t : \theta_t = \{s_0, \ldots, s_t\}, s_t \in \{\theta_t : \theta_t = \{s_0, \ldots, s_t\}, s_t \in \{\theta_t : \theta_t = \{\theta_t : \theta_t$  $\{0,1\}, t \ge 0\}$ , which is clearly countable. For the proof, we need to make a few technical assumptions: first, we require that possible beliefs over the stochastic process governing histories  $\mathbb{P}\{s_{t+1}=1|\theta_t,t\}$  are restricted to measurable functions of time and history. Second, we restrict possible values of  $\{c_t, h_t, k_t\}$  to the set  $X = [c_L, c_U] \times [h_L, h_U] \times [k_L, k_U]$ , where the lower and upper bounds satisfy:  $0 < c_L < \hat{c} < \bar{c} < c_U$  (and so on for *h* and *k*, where the 'bar' and 'hat' values are those defined in Equations (2) and (4), respectively). Thus, X is compact and convex, and all candidate equilibria discussed in this paper are in its interior. By compactness and since  $c_t = 0$  is excluded, possible values of the per-period return function  $\hat{F}(x_1, x_2) = \log(x_1) - (-x_2)^{1+\eta}/(1+\eta)$  are bounded (reading  $x_1 = c$  and  $x_2 = -h$  in the negative domain); since F is clearly a strictly concave function and the choice set allowed by the budget and health constraints (intersected with X) is convex, the household's problem now satisfies the assumptions of Theorem 9.8 by Stokey and Lucas [11], and must have a unique solution for the value function along with a unique policy function. Since, further, the reward function F is differentiable, Theorem 9.10 of [11] also applies and any interior solution of the household's problem must satisfy the usual firstorder and envelope conditions.

For the present problem, these are:

$$\begin{aligned} \frac{1}{c_t} - \lambda_t - \mu_t s_t &= 0 \\ -h_t^{\eta} + \lambda_t w_t &= 0 \end{aligned} \qquad \beta \, \mathbb{E}_t \{ \partial V(k_{t+1}, s_{t+1}) / \partial k_{t+1} \} - q_t &= 0 \\ \text{and} \quad \partial V(k_t, s_t) / \partial k_t &= \lambda_t (r_t + q_t) \end{aligned}$$

For consumption, clearly  $c_t = 1/\lambda_t$  if the health constraint is slack and  $c_t = \hat{c}$  if it binds. Rearranging the FOC for labor yields the second result. Third, iterate forward the envelope condition by one period  $(\partial V(k_{t+1}, s_{t+1})/\partial k_{t+1} = \lambda_{t+1}(r_{t+1} + q_{t+1}))$  and substitute it into the FOC for capital to obtain Equation (1).

*Proof of Proposition 2.* Suppose the pandemic is known to last for  $n \ge 0$  additional periods, that is to say, from t + 1 to t + n. We iterate Equation (1) forward:

$$\lambda_t q_t = \beta \mathbb{E}_t \Big\{ \lambda_{t+1} (r_{t+1} + q_{t+1}) \Big\} = \beta \mathbb{E}_t \Big\{ \lambda_{t+1} r_{t+1} \Big\} + \beta \mathbb{E}_t \Big\{ \lambda_{t+1} q_{t+1} \Big\}$$
$$= \dots = \mathbb{E}_t \left\{ \sum_{\tau=1}^n \beta^\tau \lambda_{t+\tau} r_{t+\tau} \right\} + \beta^n \mathbb{E}_t \Big\{ \lambda_{t+n} q_{t+n} \Big\}$$
(A.1)

Now, since we expect the economy to be in its normal steady state for all periods from period t + n + 1 on, we can evaluate Equation (1) in period t + n:

$$\mathbb{E}_t \Big\{ \lambda_{t+n} q_{t+n} \Big\} = \beta \, \mathbb{E}_t \Big\{ \lambda_{t+n+1} (r_{t+n+1} + q_{t+n+1}) \Big\} = \beta \, \bar{\lambda} (\bar{r} + \bar{q}) = \bar{\lambda} \bar{q}$$

Finally, since the pandemic is ongoing for all periods from t to t + n, we must have  $r_{t+\tau} = \alpha \hat{c}$  (via Equations 4), and  $\lambda_{t+\tau} = \lambda_t$ . Substitute these results back into (A.1):

$$\lambda_t q_t = \lambda_t \alpha \hat{c} \cdot \sum_{\tau=1}^n \beta^\tau + \beta^n \bar{\lambda} \bar{q} \quad \leftrightarrow \quad \frac{q_t}{\bar{q}} = \frac{1}{\beta \bar{c}} \sum_{\tau=1}^n \beta^\tau + \beta^n \frac{\bar{\lambda}}{\lambda_t} = (1 - \beta^n) \frac{\hat{c}}{\bar{c}} + \beta^n \left(\frac{\hat{c}}{\bar{c}}\right)^{-\frac{\eta+\alpha}{1-\alpha}}$$

where the second equality uses Equation (3) to substitute  $\bar{q}$ , and the third equality uses (2) and (4) to substitute  $\lambda_t$  and  $\bar{\lambda}$ , as well as the geometric sum formula for the  $\beta$ 's.

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