Asymmetric Information and the Liquidity Role of Assets*

Athanasios Geromichalos[†]
University of California - Davis

Lucas Herrenbrueck[‡]
Simon Fraser University

Zijian Wang[§]
Wilfrid Laurier University

February 2023

Abstract

Monetary models overwhelmingly feature a particular asset, "money", as the medium of exchange in an economy. However, the adoption of a medium of exchange is endogenous and subject to changes if conditions favor a different asset. We study the liquidity role of a real asset that is subject to asymmetric information. We find that rather than using the asset directly as a medium of exchange, agents prefer to liquidate it for money in a secondary asset market, thus establishing money as the dominant medium of exchange. Furthermore, we show that a decrease in severity of asymmetric information in the secondary asset market can hurt welfare. Finally, we find that inflation is crucial for the determination of key equilibrium variables, such as the volume of trade in the secondary asset market, and the decision of agents to invest in information that reduces the degree of information asymmetry.

Keywords: OTC markets, Adverse selection, Indirect liquidity, Medium of exchange
JEL Codes: E40, E50, G11, G12, G14

*We would like thank Lukas Altermatt, Garth Baughman, Hugo van Buggenum, Braz Camargo, Michael Choi, Pedro Gomis-Porqueras, Kee-Youn Kang, Florian Madison, Fernando Martin, Guillaume Rocheteau, Alexandros Vardoulakis, Randall Wright, Cathy Zhang, as well as audiences at the 2022 Summer Workshop in Money, Banking, Payments and Finance and the University of California Irvine for their helpful comments and suggestions. All errors are ours.

[†]ageromich@ucdavis.edu
[‡]herrenbrueck@sfu.ca
[§]zijianwang@wlu.ca

1 Introduction

Monetary models overwhelmingly feature a particular asset, "money", as the medium of exchange in an economy. Such an assumption is often empirically relevant, but it is also subject to criticism: in his famous dictum, Wallace (1998) argued that the adoption of a medium of exchange is endogenous and subject to changes if *conditions* favor a different asset. We develop a model where fiat money and a real asset compete as media of exchange, but the real asset is subject to asymmetric information and adverse selection. We find that money arises as the dominant medium of exchange, but the degree of its dominance depends on economic conditions such as access to asset markets, information frictions in these markets, as well as inflation.

In our model, agents trade periodically in a decentralized *goods market* where due to standard frictions (e.g., anonymity and limited commitment), a medium of exchange is necessary. Consistent with the Wallace Dictum, agents in our model are free to use either fiat money or the real asset as a medium of exchange. However, the producers of goods are uninformed about the quality of the real asset, which hinders its role as a (direct) medium of exchange. A novel and realistic ingredient of our model is that agents have the option to visit an over-the-counter secondary *asset market* where the real assets can be liquidated for money, and the asymmetric information concerning asset quality is typically less severe, and eventually endogenously determined.[1] Thus, in our framework, the secondary market offers agents the option to reduce the severity of the information friction: if an agent is concerned that her assets may not be accepted as a medium of exchange in the goods market, she can liquidate them and pay with money instead.

First, we consider a version of the model where the degree of asymmetric information in the asset market is exogenous. More precisely, a fraction τ of meetings operates under complete information, and we refer to them as "transparent meetings". In the remaining meetings, which we refer to as "opaque meetings", only the holder/seller of the assets knows their quality. As is standard in models with adverse selection, sellers with low-quality assets do not suffer from asymmetric information, hence they do not have an incentive to sell their assets in transparent meetings. However, high-types will sell assets in transparent meetings to prevent low-types from mimicking them in the forthcoming goods market. Interestingly, even agents who are matched in opaque meetings choose to sell assets so that they do not have to pool in the goods market with low-types who found themselves in transparent meetings. In other words, even through the asset market also suffers from asymmetric information, our framework predicts that agents are better off selling assets for money when they can (even in

[1]This captures the reasonable idea that buyers in asset markets are more likely to be informed about the quality of the assets they are about to purchase, compared to, say, a car dealer who is offered assets as payment.

opaque meetings), thus establishing money as the dominant medium of exchange.

One may expect that aggregate welfare should be increasing in τ, since more transparent asset market meetings reduce the degree of asymmetric information. Surprising, in our model, aggregate welfare is non-monotone in τ, and the intuition is as follows. A larger fraction of transparent meetings makes it more likely that agents can avoid asymmetric information by liquidating assets in the secondary market and ultimately using money as medium of exchange in the goods market. For this reason, a higher τ induces agents to reduce their money holdings, which lowers their consumption if they turn out to be low-types, and if this latter effect dominates then welfare decreases in τ.

Our model also predicts that the effect of asymmetric information on the liquidity role of assets depends on the level of inflation. When inflation is high, holding money is costly, which promotes the indirect liquidity role of assets, i.e., agents prefer to sell assets for money in the secondary market. However, when inflation is low, no agents except for those with high-quality assets in transparent meetings will sell assets in the secondary market. An interesting consequence is that while aggregate welfare is decreasing in inflation, as expected, the decrease is sometimes discontinuous. At higher levels of inflation, agents must rely more heavily on the liquidity role of assets, which exposes them to the penalty of asymmetric information. As a result, aggregate welfare can jump discontinuously as a function of inflation. This result is topical as it implies that the recent increases in inflation that economies all around the world have been experiencing may have an even larger impact on welfare than expected.

The next step is to endogenize the fraction of meetings that operate under complete information (i.e., the value of τ). Specifically, we give asset sellers the option to pay a cost κ that allows them to trade in transparent meetings. One can think of κ as the cost of producing a certificate of the asset's quality, or a fee to access an intermediary who can guarantee the quality of the asset. Assuming κ has to be paid after the quality of the assets has been realized, we find that asset sellers pay that cost if and only if inflation is high. The intuition is straightforward: when inflation is high, agents carry little money, and the value of the additional liquidity provided by the assets justifies paying the cost.

Allowing for the possibility of paying κ before the quality of the assets has been realized gives rise to more surprising results. Specifically, we find that agents only pay κ when inflation is neither too high nor too low. When inflation is low, agents can use money as (cheap) insurance against the quality shock, i.e., they use money if their assets are of low quality. In that case, agents do not need to rely heavily on asset liquidity, so they prefer not to pay κ. But why would agents not pay this cost when inflation is high and money holdings are scarce? This is because opaque meetings allow agents to obtain liquidity from assets even if they receive the bad quality shock. In that sense, opaque meetings also serve as insurance against the quality shock, which is especially valuable to agents when their money holdings are low. Thus, allowing for an endogenous determination of τ highlights new, important insights regarding the liquidity role provided by assets, as money is no longer the unique form of insurance against the quality shock. Consequently, agents choose not to pay κ and trade in opaque meetings, precisely when the cost of the alternative form of insurance (money) is especially high, i.e., when inflation is high.

Finally, we examine how the size of the information cost κ affects aggregate welfare. Surprisingly, we find that as long as inflation is relatively low, the aggregate welfare can be increasing in κ, and this is independent of when κ is paid. Intuitively, a large κ discourages agents from investing in information to participate in transparent meetings, which diminishes the liquidity role of assets. When inflation is low, agents do not pay κ (i.e., equilibrium $\tau = 0$), and assets do not provide liquidity services because agents carry large amounts of real balances. This ultimately increases agents' consumption if they turn out to be low-types.

The model in this paper is based on Geromichalos and Herrenbrueck (2016), who extend the Lagos and Wright (2005) framework by introducing an over-the-counter asset market to allow agents to rebalance their portfolios.[2] The present paper improves upon this framework by removing the cash-in-advance constraint, thereby providing micro-foundations to why money is the superior medium of exchange and why agents trade in OTC markets. The closest work that studies the role of private information in asset acceptability is Rocheteau (2011). In that paper, money and assets can serve as media of exchange in a decentralized goods market, where producers cannot observe the quality of the assets, which impairs their liquidity role. Our model adopts a similar framework, however agents have access to a secondary asset market where the degree of asymmetric information is less severe (see Footnote 1). By doing so, our model not only introduces an empirically-relevant concept of asset liquidity (i.e., assets can be sold for money in a secondary market) but also allows us to study how the degree of asymmetric information in the secondary market affects asset liquidity and aggregate welfare.

In other related work, Madison (2019) and Wang (2020) study how asymmetric information affects the indirect liquidity of assets. Geromichalos, Jung, Lee, and Carlos (2021) also study the coexistence of direct and indirect asset liquidity, but they do not directly deal with asymmetric information, because they assume that producers in decentralized goods market never accept assets they do not recognize. Lu (2022) studies asset liquidity under the assumption that buyers of assets, instead of sellers, possess more information. Lester, Postlewaite,

[2]This concept of "indirect liquidity" is also studied by Berentsen, Huber, and Marchesiani (2014), Mattesini and Nosal (2016), Geromichalos, Herrenbrueck, and Salyer (2016), Herrenbrueck and Geromichalos (2017), Herrenbrueck (2019), Geromichalos and Herrenbrueck (2022), and Altermatt, Iwasaki, and Wright (2022). Other papers that explore the idea of rebalancing asset portfolios include Kocherlakota (2003), Boel and Camera (2006), Berentsen, Camera, and Waller (2007), and Berentsen and Waller (2011). None of these papers study private information.

and Wright (2012) develop a framework where the degree of asset recognizability affects the degree of asset acceptability. Finally, our paper belongs to the vast literature on asymmetric information spurred by Akerlof (1970) and Leland and Pyle (1977). Some recent work that also studies asymmetric information in asset markets includes Eisfeldt (2004), Kurlat (2013), Guerrieri and Shimer (2014), Chiu and Koeppl (2016), and Choi (2018).[3]

The rest of the paper is organized as follows. Section 2 introduces the model. In Section 3, we describe the equilibrium with an exogenous τ, while we endogenize τ in Section 5. Finally, Section 6 concludes the paper.

2 Environment

Time is discrete and continues forever. Each period is divided into three subperiods: the asset market (AM), the decentralized market (DM), and the centralized market (CM). There is measure one of "consumers" and measure one of "producers", named for their roles in the consumption and production of DM goods (q). In the CM, there is a single good (x) that can be produced and consumed by all agents, and which also serves as the numéraire. A consumer's instantaneous utility is given by:

$$\eta_t u(q_t) + x_t, \tag{2.1}$$

where q_t and x_t are the consumption of the DM good and the CM good, respectively. We assume $q_t \geq 0$, but x_t can be negative, in which case it is interpreted as production in the CM: one unit of labor in the CM can be turned into one unit of CM good. We assume η_t is stochastic. Specifically,

$$\eta_t = \begin{cases} 0, & \text{with probability } 1-\lambda; \\ 1, & \text{with probability } \lambda. \end{cases} \tag{2.2}$$

If $\eta_t = 0$, a consumer does not derive utility from the DM good and is referred to as an "N-type" consumer. If $\eta_t = 1$, a consumer derives utility from the DM good and is referred to as a "C-type" consumer. Consumers learn the value of η_t at the beginning of the AM. The realization of η_t is *i.i.d.* across consumers and time, and whether a consumer is a C-type (i.e., $\eta_t = 1$) or an N-type (i.e., $\eta_t = 0$) is common knowledge. We assume that $u(0) = 0$, $u'(.) > 0$, $u''(.) < 0$, $u'(0) = \infty$. We also define the first-best quantity of DM good as q^*,

[3] Other work that studies private information in asset markets includes Williamson and Wright (1994), Trejos (1997), Trejos (1999), Velde, Weber, and Wright (1999), Li, Rocheteau, and Weill (2012), Lester et al. (2012), Gorton and Ordonez (2014), Golosov, Lorenzoni, and Tsyvinski (2014), Camargo and Lester (2014), Chari, Shourideh, and Zetlin-Jones (2014), Carapella and Williamson (2015), Lauermann and Wolinsky (2016), Ozdenoren, Yuan, and Zhang (2019), and Cai and Dong (2020).

where $u'(q^*) = 1$. Next, the instantaneous utility of a producer is given by

$$-h_t + X_t, \tag{2.3}$$

where h_t is the amount of labor supplied in the DM, and X_t is the consumption of the CM good. Similar to x_t, X_t can be negative, in which case it is interpreted as production in the CM. We assume one unit of labor in the DM can be turned into one unit of the DM good. Neither goods can be carried across periods. All agents discount future utility using $\beta \in (0, 1)$.

There are two types of assets in the economy: (fiat) money and perfectly divisible real assets.[4] Money is issued by a government, and each consumer is endowed with a units of the real assets in each CM. Following Rocheteau (2011), we interpret the real asset as private equity, corporate bonds, or asset-backed securities. In the CM of the next period, each unit of assets produces a dividend of δ units of CM good before depreciating by 100%. Again in line with Rocheteau (2011), we assume that δ is stochastic, and that by holding the real assets, consumers learn their quality.[5] Specifically, at the beginning of the AM, with probability ρ, a consumer learns that her real assets have high quality, and each unit will produce $\delta = \delta_h > 0$ units of the CM good. With probability $1 - \rho$, a consumer learns that her real assets have low quality, and each unit of the assets will produce $\delta = \delta_l \geq 0$ units of the CM good. We study $\delta_l = 0$ as a separate case, and we interpret it as the case where asset holders have received private information that the asset is either fake or will default in the following CM. Finally, we assume that the realization of δ is independent across consumers and is independent of the realization of η_t.

We assume agents are anonymous. Therefore, a medium of exchange is necessary for the trade in the DM. We assume both money and real assets can be used as payment instruments. In addition, consumers can trade their assets in the asset market. Specifically, in the AM, after consumers learn η_t and the quality of their assets, a market opens for asset trade. An asset seller is randomly matched with an asset buyer. We assume that a fraction, τ, of the meetings are "transparent" in the sense that the buyers can observe the quality of the real assets. The remaining $1 - \tau$ of the meetings are "opaque" in the sense that the quality of the assets, δ, is asset sellers' private information. Once matched, the asset seller makes a take-it-or-leave-it offer to the asset buyer. The offer consists of a unit price, ψ, and a quantity for sale, s. If an offer is accepted, the seller receives ψs units of money, and the buyer receives s units of assets. To simplify the analysis, we assume each asset seller is matched with a buyer with probability one.

[4]The analysis would remain unaltered if the assets were nominal.

[5]Plantin (2009) shows that this assumption is of particular relevance for assets like collateralized debt obligations and privately placed debt, which are securities sold to selected investors and are bundled with future access to privileged information about the assets. Similar assumptions can be found in Rocheteau (2011), Madison (2017, 2019), and Wang (2020).

After the AM, each C-type consumer is matched with a producer with probability one in the DM, and C-type consumers make take-it-or-leave-it offers to producers. We assume that both assets and money can be used as means of payment. However, the quality of the assets is consumers' private information in all meetings between C-type consumers and producers.

Finally, in the CM, agents produce, trade, and consume the CM good. Agents also choose how much money to bring to the next period. Since our focus is the trade in the AM and DM, for simplicity, we assume agents do not trade the real assets in the CM. Lastly, let M_t denote the supply of money. It satisfies:

$$M_{t+1} = (1+\mu)M_t, \tag{2.4}$$

where μ is the money growth rate. We assume $\mu > \beta - 1$. Money is injected to (or withdrawn from) the economy by a monetary authority via a lump-sum transfer (or tax) in each CM.

3 Definition of Equilibrium

In this section, we describe the agents' problems and define the equilibrium. It turns out to be convenient to start with the DM: consider a C-type consumer with $\tilde{z}$ units of money (in real terms) and $\tilde{a}$ units of assets. Once she is matched with a producer, the C-type consumer makes a take-it-or-leave-it offer $(\hat{q}, \hat{z}, \hat{a})$ to the producer, where $\hat{q}$ is the quantity of goods she wants to purchase, and $\hat{z}$ and $\hat{a}$ are the amount of money and the quantity of assets the producer will receive, respectively. Since asset quality is the consumer's private information, the producer must form beliefs regarding asset quality. Let $\gamma^g(\hat{q}, \hat{z}, \hat{a}; \tilde{z}, \tilde{a})$ denote the producer's belief about the probability of the asset quality being high (i.e., $\delta = \delta_h$), conditional on the consumer's offer $(\hat{q}, \hat{z}, \hat{a})$ and asset portfolio $(\tilde{z}, \tilde{a})$.

As is standard in the Lagos and Wright (2005) literature, both the consumer's and producer's value functions in the DM are linear in money and assets, and the price of the DM good in terms of the CM good is one. The consumer solves the following problem.

$$\max_{\hat{q},\hat{z},\hat{a}} \{u(\hat{q}) - \hat{z} - \delta\hat{a}\} \tag{3.1}$$

$$\text{s.t. } [\gamma^g(\hat{q}, \hat{z}, \hat{a}; \tilde{z}, \tilde{a})\delta_h + (1 - \gamma^g(\hat{q}, \hat{z}, \hat{a}; \tilde{z}, \tilde{a})\delta_l]\hat{a} + \hat{z} \geq \hat{q}, \tag{3.2}$$

$$\hat{z} \leq \tilde{z}, \;\; \hat{a} \leq \tilde{a}. \tag{3.3}$$

In words, problem (3.1) says that the consumer receives $u(\hat{q})$ from consuming the DM good, but she has to give up $\hat{z}$ units of money and $\hat{a}$ units of assets. For the producer to accept the offer, her expected value of the assets, $[\gamma^g(\hat{q}, \hat{z}, \hat{a}; \tilde{z}, \tilde{a})\delta_h + (1 - \gamma^g(\hat{q}, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}))\delta_l]\hat{a}$, plus the value of money must at least compensate for the cost of producing the DM good, $\hat{q}$.

Next, we turn to agents' problems in the AM. Recall that only consumers participate in the AM. Let z_s and a denote the units of money and assets held by a C-type consumer, respectively. Let z_b denote the units of money held by an N-type consumer. We restrict our attention to the case where $z_s < q^*$. We will show later that this always holds as long as $\mu > \beta - 1$. This ensures that if a consumer does not trade in the AM, using only money will not allow her to consume the efficient amount in the DM. This gives C-type consumers the incentive to either use assets directly in the DM or trade them for money in the AM.

Recall that whether a consumer is a C-type (i.e., $\eta_t = 1$) or an N-type (i.e., $\eta_t = 0$) is common knowledge. This means that assets sellers can only be C-type consumers and asset buyers can only be N-type consumers, since there is no surplus to be gained from asset trade between two C-type consumers or two N-type consumers. Once a seller is matched with a buyer, the seller makes a take-it-or-leave-it offer (ψ, s) to the buyer, where ψ is the price of the asset, and s is the quantity of the asset for sale.

Now, define $q(\tilde{z}, \tilde{a}, \delta, \gamma^g)$ to be the solution to problem (3.1) conditional on a C-type consumer's portfolio at the beginning DM $(\tilde{z}, \tilde{a})$, the quality of the consumer's assets δ, and producers' beliefs γ^g. First, let us consider an asset seller in transparent meetings. If her offer is accepted by the buyer, she enters DM with $z_s + \psi s$ units of money and $a - s$ units of assets. To determine (ψ, s), she solves the following problem.

$$\max_{\psi,s}\{u(q(z_s + \psi s, a - s, \delta_h, \gamma^g)) - u(q(z_s, a, \delta_h, \gamma^g)) - \delta s\} \tag{3.4}$$

$$\text{s.t. } \psi \leq \delta,\ \psi s \leq z_b,\ s \leq a. \tag{3.5}$$

In words, if the offer is accepted, the seller obtains a surplus equal to $u(q(z_s+\psi s, a-s, \delta_h, \gamma^g)) - u(q(z_s, a, \delta_h, \gamma^g))$ but has to give up s units of assets, which will generate δs of CM goods in the CM. In addition, for the offer to be accepted by the asset buyer, the unit price of the asset cannot be higher than the dividend per unit of assets.

Finally, consider asset sellers in opaque meetings. Since asset quality is seller's private information, buyers must form beliefs regarding asset quality conditional on the offers. Let $\gamma^a(\psi, s)$ denote a buyer's belief about the probability of the asset quality being high (i.e., $\delta = \delta_h$ conditional on the seller's offer. A seller solves the following problem.

$$\max_{\psi,s}\{u(q(z_s + \psi s, a - s, \delta_h, \gamma^g)) - u(q(z_s, a, \delta_h, \gamma^g)) - \delta s\} \tag{3.6}$$

$$\text{s.t. } \psi \leq (1 - \gamma^a(\psi, s))\delta_l + \gamma^a(\psi, s)\delta_h,\ \psi s \leq z_b,\ s \leq a. \tag{3.7}$$

Compared to problem (3.4), the only difference is that the asset price cannot exceed $(1 - \gamma^a(\psi, s))\delta_l + \gamma^a(\psi, s)\delta_h$, the buyer's expected amount of dividend per unit of assets.

There are four kinds of C-type consumers in the AM and the DM depending on the quality of their assets and the types of their AM meetings. We denote these C-type consumers as type-ij consumers, where $i \in \{l, h\}$ stands for the quality of their assets (l for low quality and h for high quality), and $j \in \{T, O\}$ stands for the AM meeting type (T for transparent and O for opaque). Now, we define the Perfect Bayesian Equilibrium in the AM and the DM.

Definition 1 *Conditional on (z_s, z_b, a), a Perfect Bayesian Equilibrium in the AM and the DM consists of offers from consumers $\{(\psi^{jk}, s^{jk}; \hat{q}^{jk}, \hat{z}^{jk}, \hat{a}^{jk})\}$ where $j \in \{l, h\}$ and $k \in \{O, T\}$, a decision rule by asset buyers $\mathbb{1}^a(\psi, s)$, a belief function by asset buyers $\gamma^a(\psi, s)$, a decision rule by producers $\mathbb{1}^g(\hat{q}, \hat{z}, \hat{a})$, a belief function by producers $\gamma^g(\hat{q}, \hat{z}, \hat{a}; \tilde{z}, \tilde{a})$ such that*
(1) (ψ^{jT}, s^{jT}) solves (3.4) for $j \in \{l, h\}$; (ψ^{jO}, s^{jO}) solves (3.6) for $j \in \{l, h\}$; $\{(\hat{q}^{jk}, \hat{z}^{jk}, \hat{a}^{jk})\}$ solves (3.1) for $j \in \{l, h\}$ and $k \in \{O, T\}$.
(2) $\mathbb{1}^a(\psi, s) = 1$ if and only if $\psi \le \gamma^a(\psi, s)\delta_h + (1 - \gamma^a(\psi, s))\delta_l$; $\mathbb{1}^g(\hat{q}, \hat{z}, \hat{a}) = 1$ if and only if $[\gamma^g(\hat{q}, \hat{z}, \hat{a}; \tilde{z}, \tilde{a})\delta_h + (1 - \gamma^g(\hat{q}, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}))\delta_l]a + z \ge q$.
(3) $\gamma^a(\psi, s)$ and $\gamma^g(\hat{q}, \hat{z}, \hat{a}; \tilde{z}, \tilde{a})$ are derived from Bayes' rule whenever possible.

Conditions (1) and (2) guarantee that agents' strategies are optimal given asset buyers' and producers' beliefs. In the following proposition, we describe the full set of Perfect Bayesian Equilibria in AM and DM.

Proposition 1 *Given (z_s, z_b, a), the full set of Perfect Bayesian Equilibria in the AM and the DM consists of offers $\{(\psi^{jk}, s^{jk}; \hat{q}^{jk}, \hat{z}^{jk}, \hat{a}^{jk})\}$ where $j \in \{l, h\}$ and $k \in \{O, T\}$ and belief functions $\gamma^a(\psi, s)$ and $\gamma^g(\hat{q}, \hat{z}, \hat{a}; \tilde{z}, \tilde{a})$ that satisfy the following:*
(1) For all j and k, $\psi^{jk} \le \gamma^a(\psi^{jk}, s^{jk})\delta_h + (1 - \gamma^a(\psi^{jk}, s^{jk}))\delta_l$, and $[\gamma^g(\hat{q}^{jk}, \hat{z}^{jk}, \hat{a}^{jk}; \tilde{z}, \tilde{a})\delta_h + (1 - \gamma^g(\hat{q}^{jk}, \hat{z}^{jk}, \hat{a}^{jk}; \tilde{z}, \tilde{a}))\delta_l]\hat{a}^{jk} + \hat{z}^{jk} \ge \hat{q}^{jk}$, where $\tilde{z} = z_s + \psi^{jk}s^{jk}$ and $\tilde{a} = a - s$.
(2) Define $\tilde{v}^{jO}(\tilde{z}, \tilde{a})$ to be C-type consumers' DM surplus with DM portfolio $(\tilde{z}, \tilde{a})$

$$\tilde{v}^{jO}(\tilde{z}, \tilde{a}) \equiv \max_{\hat{q}, \hat{z} \le \tilde{z}, \hat{a} \le \tilde{a}} \{u(\hat{q}) - \hat{z} - \delta\hat{a}\} \text{ s.t. } [\gamma^g(\hat{q}, \hat{z}, \hat{a}; \tilde{z}, \tilde{a})\delta_h + (1 - \gamma^g(\hat{q}, \hat{z}, \hat{a}; \tilde{z}, \tilde{a})\delta_l]\hat{a} + \hat{z} \ge \hat{q}.$$

Define $\underline{v}^{jO}$ to be C-type consumers' lowest possible surplus in the AM

$$\underline{v}^{jO} \equiv \max_{\psi, s} \{\tilde{v}^{jO}(z_s + \psi s, a - s) - \tilde{v}^{jO}(z_s, a) - \delta s\} \text{ s.t. } \psi \le \delta_l,\ \psi s \le z_b,\ s \le a.$$

Then, $\gamma^a(\psi^{jO}, s^{jO})$ is given by

$$\gamma^a(\psi^{jO}, s^{jO}) = \begin{cases} 1, \text{ if } \tilde{v}^{lO}(z^\dagger + \psi^{jO}s^{jO}, a^\dagger - s^{jO}) \le \underline{v}^{lO} \text{ and } \tilde{v}^{hO}(z^\dagger + \psi^{jO}s^{jO}, a^\dagger - s^{jO}) \ge \underline{v}^{hO}; \\ \rho, \text{ if } \tilde{v}^{lO}(z^\dagger + \psi^{jO}s^{jO}, a^\dagger - s^{jO}) > \underline{v}^{lO} \text{ and } \tilde{v}^{hO}(z^\dagger + \psi^{jO}s^{jO}, a^\dagger - s^{jO}) \ge \underline{v}^{hO}; \\ 0, \text{ if } \tilde{v}^{lO}(z^\dagger + \psi^{jO}s^{jO}, a^\dagger - s^{jO}) \ge \underline{v}^{lO} \text{ and } \tilde{v}^{hO}(z^\dagger + \psi^{jO}s^{jO}, a^\dagger - s^{jO}) < \underline{v}^{hO}, \end{cases} \tag{3.8}$$

while $\gamma^a(\psi^{jT}, s^{jT}) = 1$ *if and only if* $j = h$.
(3) *Define* $\tilde{\psi} = \frac{\tilde{z}-z_s}{a-\tilde{a}}$ *to be the price of assets sold in the AM. Let* $\tilde{\psi} = 0$ *if* $(\tilde{z}, \tilde{a}) = (z_s, a)$. *Define* $\underline{\tilde{v}}^j(\tilde{z}, \tilde{a})$ *to be C-type consumers' lowest possible surplus in the DM*

$$\underline{\tilde{v}}^j(\tilde{z}, \tilde{a}) \equiv \max_{\hat{q},\hat{z},\hat{a}} \{u(\hat{q}) - \hat{z} - \delta_j \hat{a}\} \text{ s.t. } \delta_l \hat{a} + \hat{z} \geq \hat{q},\ \hat{z} \leq \tilde{z},\ \hat{a} \leq \tilde{a}.$$

Then, $\gamma^g(\hat{q}, \hat{z}, \hat{a}; \tilde{z}, \tilde{a})$ *is given by*

$$\gamma^g(\hat{q}, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}) = \begin{cases} 1, & \text{if } \ u(\hat{q}) - \hat{z} - \delta_l \hat{a} \leq \underline{\tilde{v}}^l(\tilde{z}, \tilde{a}) \ \text{ and } \ u(\hat{q}) - \hat{z} - \delta_h \hat{a} \geq \underline{\tilde{v}}^h(\tilde{z}, \tilde{a}), \\ & \text{or if } \ \tilde{\psi} \in (\rho\delta_h + (1-\rho)\delta_l, \delta_h]; \\ \rho, & \text{if } \tilde{\psi} \in (\delta_l, \rho\delta_h + (1-\rho)\delta_l]; \\ 0, & \text{if otherwise.} \end{cases} \tag{3.9}$$

(4) $\gamma^a(\psi, s) = 0$ *and* $\gamma^g(\hat{q}, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}) = 0$ *for all* $(\psi, s; \hat{q}, \hat{z}, \hat{a}) \notin \{(\psi^{jk}, s^{jk}; \hat{q}^{jk}, \hat{z}^{jk}, \hat{a}^{jk})\}$.
Proof: See Appendix B.

Condition (1) ensures that it is optimal for asset buyers in the AM and producers in the DM to accept C-type consumers' offers conditional on asset buyers' and producers' beliefs. Condition (2) guarantees that asset buyers' beliefs on the equilibrium path, $\gamma^a(\psi^{jk}, s^{jk})$, are consistent with C-type consumers' strategies in the AM. Specifically, asset buyers believe that an offer comes from a C-type with quality-j assets, $j \in \{l, h\}$, if and only if the C-types with quality-j assets have the incentive to make such an offer. In the AM, the offers can either be pooling (i.e., $(\psi^{lO}, s^{lO}) = (\psi^{hO}, s^{hO})$) or separating (i.e., $(\psi^{lO}, s^{lO}) \neq (\psi^{hO}, s^{hO})$). Condition (3) requires $\gamma^g(\hat{q}, \hat{z}, \hat{a}; \tilde{z}, \tilde{a})$ to be consistent with C-type consumers' strategies in the DM:
a. Similar to condition (2), producers believe that an offer comes from a C-type with high-quality assets if and only if C-types with high-quality assets are the sole type that have the incentive to make such an offer.
b. Producers can also infer from consumers' portfolios the quality of their assets:

(i) Because asset buyers' beliefs must satisfy Bayes' rule, in the AM only C-types with high-quality assets (either in transparent meetings or opaque meetings) may sell at a price that is strictly higher than $\rho\delta_h + (1-\rho)\delta_l$. To see why, note that C-types with low-quality assets in transparent meetings can at most sell at $\psi = \delta_l$. Next, if C-types in opaque meetings make pooling offers, they can at most sell at $\psi = \rho\delta_h + (1-\rho)\delta_l$, which is the average asset quality in opaque meetings. If C-types in opaque meetings make separating offers, those with low-quality assets can at most sell at $\psi = \delta_l$.

(ii) The belief in (i) ensures that C-types with high-quality assets in transparent meetings will not sell at price less that $\rho\delta_h + (1-\rho)\delta_l$. This in turn ensures that only C-types in opaque meetings may sell at a price that is strictly higher than δ_l but less than $\rho\delta_h + (1-\rho)\delta_l$, which only happens when they make pooling offers.

(iii) The belief in (ii) ensures that C-types in opaque meetings, when making pooling offers, will not sell at price less that δ_l. Hence, if $\tilde{\psi} \leq \delta_l$, then the consumer must have low-quality assets.

Finally, condition (4) is a sufficient but not necessary condition on off-equilibrium path beliefs that ensures no consumers have the incentive to deviate and make an offer not in the set of equilibrium offers $\{(\psi^{jk}, s^{jk}; \hat{q}^{jk}, \hat{z}^{jk}, \hat{a}^{jk})\}$.

Proposition 1 shows that a plethora of equilibria exist under Definition 1, because no restrictions are put on off-equilibrium path beliefs. To conduct meaningful analysis, equilibrium refinement is necessary. In Definition 2, we adapt the *undefeated equilibrium refinement* proposed by Mailath, Okuno-Fujiwara, and Postlewaite (1993) to our environment.[6]

Definition 2 *A Perfect Bayesian Equilibrium in the AM and the DM (PBE-1) is defeated by another Perfect Bayesian Equilibrium in the AM and the DM (PBE-2) if there exists* $(\hat{\psi}', \hat{s}'; \hat{q}', \hat{z}', \hat{a}')$ *that satisfies*

(1) *There exist* $K \subseteq \{lO, hO, lT, hT\}$ *such that* $(\hat{\psi}', \hat{s}'; \hat{q}', \hat{z}', \hat{a}')$ *is played by type-*K *consumers in PBE-2 but not in PBE-1.*

(2) *There exist* $J \subseteq K$ *such that type-*J *consumers play* $(\hat{\psi}, \hat{s}; \hat{q}, \hat{z}, \hat{a})$ *and obtain strictly higher surplus in PBE-2.*

(3) *In PBE-1, at least one of the following conditions is satisfied:*

$$\gamma^a(\hat{\psi}', \hat{s}') \neq \frac{\rho \mathbb{1}(hO \in J)}{\rho \mathbb{1}(hO \in K) + (1-\rho)\mathbb{1}(lO \in K)}, \tag{3.10}$$

$$\gamma^g(\hat{q}', \hat{z}', \hat{a}'; z_s + \hat{\psi}'\hat{s}', a_s - \hat{s}') \neq \frac{\rho(1-\tau)\mathbb{1}(hO \in J) + \rho\tau\mathbb{1}(hT \in J)}{\rho(1-\tau)\mathbb{1}(hO \in K) + (1-\rho)(1-\tau)\mathbb{1}(lO \in K) + \rho\tau\mathbb{1}(hT \in K) + (1-\rho)\tau\mathbb{1}(lT \in K)}. \tag{3.11}$$

A Perfect Bayesian Equilibrium in the AM and the DM is undefeated if and only if there does not exist $(\hat{\psi}', \hat{s}'; \hat{q}', \hat{z}', \hat{a}')$ *that satisfies conditions* (1)-(3).

In words, Definition 2 requires a Perfect Bayesian Equilibrium to be undefeated in the sense that there does not exist a deviation that satisfies (1) it is part of another Perfect Bayesian Equilibrium; (2) it benefits some C-types; and (3) asset buyers and/or producers in the original Perfect Bayesian Equilibrium fail to anticipate such a profitable deviation. The main benefit of using the undefeated equilibrium refinement in our environment, especially when

[6]Undefeated equilibrium refinement has also been used in other papers that study asymmetric information in asset markets. See for example Rocheteau (2008, 2011), Bajaj (2018), Madison (2017), and Wang (2020).

compared to the popular Intuitive Criterion refinement (Cho and Kreps, 1987), is that it allows pooling equilibria (i.e., C-types with low-quality and high-quality assets to make the same offer in the AM and/or the DM) to exist as long as they are Pareto-optimal. However, we cannot use the definition in Mailath et al. (1993) "off the shelf", because in that paper, trading happens between two parties; in our paper, both asset buyers *and* producers must form beliefs regarding the quality of the asset. In Appendix A, we explain in detail (a) the logic behind the Undefeated Equilibrium refinement and why we use it instead of the Intuitive Criterion; and (b) how we adapt the Undefeated Equilibrium refinement of Mailath et al. (1993) to our environment and why the version in this paper captures the logic of the original version.

Definitions 1 and 2 define undefeated equilibria in the AM and the DM, taking as given consumers' asset portfolios. Recall that in each CM, each consumer is endowed with a unit of the real asset, but their money holdings for the following period are determined endogenously. In the following section, we characterize undefeated equilibria in the AM and the DM as well as consumers' optimal choice of z.

4 Characterization of Equilibrium

4.1 The case where $\delta_l = 0$

We start with the case where $\delta_l = 0$. One can think of this case as a scenario where the asset holders receive private information that the asset is counterfeit or will default in the following CM. Define the variable $\tilde{q}$ as the solution to $u'(\tilde{q}) = 1/\rho$. The following proposition describes undefeated equilibria in the AM and the DM, taking as given consumers' portfolios (z_s, z_b, a).

Proposition 2 *Conditional on (z_s, z_b, a), undefeated equilibria in the AM and the DM satisfy the following conditions.*
(1) $(\psi^{hO}, s^{hO}; \hat{q}^{hO}, \hat{z}^{hO}, \hat{a}^{hO}) = (\psi^{lO}, s^{lO}; \hat{q}^{lO}, \hat{z}^{lO}, \hat{a}^{lO}) = (\psi_p, s_p, \hat{q}_p; \hat{z}_p, \hat{a}_p)$, *where*

(a) $\psi_p = \rho\delta_h$; (b) $s_p = (0, \min\{z_b/(\rho\delta_h), (\hat{q}_p - z_s)/(\rho\delta_h), a\}]$;

(c) $\hat{q}_p = \max\{z_s, \min\{z_s + \rho\delta_h a, \tilde{q}\}\}$; (d) $\hat{z}_p = z_s + \psi_p s_p$; (e) $\hat{a}_p = \frac{\hat{q}_p - \hat{z}_p}{\rho\delta_h}$.

(2) $(\psi^{hT}, s^{hT}; \hat{q}^{hT}, \hat{z}^{hT}, \hat{a}^{hT})$ *is given by*

(a) $\psi^{hT} = \delta_h$; (b) $\hat{q}^{hT} = \min\{q^*, z_s + \delta_h a\}$;

(c) $s^{hT} = (0, \min\{z_b/\delta_h, a\}]$; (d) $\hat{z}^{hT} = z_s + \psi^{hT} s^{hT}$; (e) $\hat{a}^{hT} = \frac{\hat{q}^{hT} - \hat{z}^{hT}}{\delta_h}$.

(3) $(\psi^{lT}, s^{lT}; \hat{q}^{lT}, \hat{z}^{lT}, \hat{a}^{lT}) = (0, 0; z_s, z_s, 0)$.

Proof: See Appendix B.

The proposition says that there are three different offers being made in the DM. By selling assets in the AM, type-hT, type-lO and type-hO consumers differentiate themselves from type-lT consumers, who are the only type that cannot obtain money from the AM. The existence of the AM allows type-hT to avoid information asymmetry and obtain the allocations under complete information. Conditional on the existence of transparent meetings, type-lO and type-hO consumers also benefit from trading in the AM because it allows them to prevent type-lT consumers from mimicking their offers. However, because type-lO and type-hO make pooling offers in both the AM and the DM, the price for the asset is discounted due to information asymmetry. If $z \geq \tilde{q}$, for type-hO consumers, the marginal benefit of using assets to either obtain money in the AM or purchase goods directly in the DM exceeds to the cost. Consequently, type-hO consumers only use money in the DM and do not trade assets in the AM. Type-lO consumers always follow type-hO consumers' strategy, because otherwise they would be identified as having low-quality assets.

Next, consider consumers' choice of real balances in the CM. We focus on symmetric solutions, so $z_s = z_b = z$. Based on the equilibria in the AM and DM, consumers solve the following problem in the CM.

$$\max_z \lambda\{(1-\rho)\tau[u(\hat{q}^{lT}) + z - \hat{q}^{lT}] + \rho\tau[u(\hat{q}^{hT}) + z + \delta_h a - \hat{q}^{hT}] \\ + (1-\tau)[u(\hat{q}_p) + z + \rho\delta_h a - \hat{q}_p]\} + (1-\lambda)(z + \rho\delta_h a) - \frac{(1+\mu)z}{\beta}, \tag{4.1}$$

where $\hat{q}^{lT}$, $\hat{q}^{hT}$ and $\hat{q}_p$ are given by Proposition 2. In words, the problem says that with probability λ, a consumer becomes a C-type consumer. With probability $(1-\rho)\tau$, the C-type consumer has low quality assets and is in a transparent meeting in the AM. With probability $\rho\tau$, the C-type consumer has high quality assets and is in a transparent meeting in the AM. Finally, with probability $1-\tau$, the C-type consumer is in an opaque meeting in the AM. Recall that in this case, consumers with high-quality assets and consumers with low-quality assets make the same offers in both the AM and the DM. With probability $1-\lambda$, a consumer becomes an N-type consumer. Note that the expected AM surplus of an N-type consumer is zero. The cost of accumulating z units of real balances in the last CM is $\frac{(1+\mu)z}{\beta}$.

The first-order condition is

$$\frac{1+\mu}{\beta} - 1 = \lambda\{(1-\rho)\tau[u'(\hat{q}^{lT}) - 1] + \rho\tau[u'(\hat{q}^{hT}) - 1] \\ + (1-\tau)[u'(\hat{q}_p)\mathbf{1}(z > \tilde{q} \text{ or } z + \rho\delta_h a < \tilde{q}) + \mathbf{1}(\tilde{q} \leq z + \rho\delta_h a \leq \tilde{q} + \rho\delta_h a) - 1]\}, \tag{4.2}$$

From Proposition 2, we know that in both the AM and DM, the assets of type-hO consumers are sold at a discount. Therefore, as long as type-hO consumers sell assets in the AM and/or use assets as medium of exchange in the DM, the marginal utility of consumption, $u'(\hat{q}_p)$, cannot be lower than the marginal cost, $1/\rho$. This means that if $z + \rho\delta_h a \geq \tilde{q}$ where $\tilde{q}$ solves $u'(\tilde{q}) = 1/\rho$, for type-lO and type-hO consumers, having more money at the beginning of the AM will not lead to higher consumption in the DM unless $z > \tilde{q}$. If $z > \tilde{q}$, type-lO and type-hO consumers will not sell assets in the AM, and they use only money to purchase the DM good, since the marginal utility, $u'(z)$, is already lower than the marginal cost of using assets as payment, $1/\rho$. This means that, if $\tilde{q} \leq z + \rho\delta_h a \leq \tilde{q} + \rho\delta_h a$, the marginal value of real balances in the next period is equal to one.

To solve the problem, first we consider the case where $z+\rho\delta_h a < \tilde{q}$. In such case, $\hat{q}^{lT} = z$, $\hat{q}^{hT} = \max\{z+\delta_h a, q^*\}$, and $\hat{q}_p = z+\rho\delta_h a$. Define $i \equiv \frac{1+\mu}{\beta} - 1$ and $\tilde{p} = \frac{p}{\beta} - \rho\delta_h$. The first-order condition becomes:

$$i = \lambda\{(1-\rho)\tau[u'(z)-1] + \rho\tau[u'(\min\{z+\delta_h a, q^*\})-1] + (1-\tau)[u'(z+\rho\delta_h a)-1]\}, \quad (4.3)$$

It is clear that there exists a unique z that solves (4.3). Now let us consider other cases. If $\tilde{q} \leq z + \rho\delta_h a \leq \tilde{q} + \rho\delta_h a$, the first order conditions become:

$$i = \lambda\{(1-\rho)\tau[u'(z)-1] + \rho\tau[u'(\min\{z+\delta_h a, q^*\})-1]\}, \quad (4.4)$$

If $z > \tilde{q}$, the first order condition becomes:

$$i = \lambda\{(1-\rho)\tau[u'(z)-1] + \rho\tau[u'(\min\{z+\delta_h a, q^*\})-1] + (1-\tau)[u'(z)-1]\}, \quad (4.5)$$

The following proposition summarizes the solution to the portfolio problem in the CM.

Proposition 3 *Suppose $\rho\delta_h a < \tilde{q}$. Then, given a, there exist $i_1 > i_2 \geq i_3 > 0$ such that*
(1) *for all $i \geq i_1$, z solves* (4.3)*;* (2) *for all $i_2 \leq i < i_1$, $z = \tilde{q} - \rho\delta_h a$;*
(3) *for all $i_3 < i < i_2$, z solves* (4.4)*;* (4) *for all $i \leq i_3$, z solves* (4.5)*.*
If $\rho\delta_h a \geq \tilde{q}$, there exists $i^\dagger$ such that
(1) *for all $i > i^\dagger$, z solves* (4.4)*;* (2) *for all $i \leq i^\dagger$, z solves* (4.5)*.*
Proof: See Appendix B.

Depending on parameter values, there exist either two or three cutoff values of i (see Figure 1-3 for an example). When $i \geq i_1$, the cost of holding real balances is high, so z is sufficiently small, and type-lO and type-hO consumers are constrained in the DM. Next, note that the marginal value of real balances is smaller than i_2 for all $z > \tilde{q} - \rho\delta_h a$ but larger than i_1 for all $z < \tilde{q} - \rho\delta_h a$. Hence, $z = \tilde{q} - \rho\delta_h a$ if $i_2 \leq i < i_1$. When $i_3 < i < i_2$, a larger money holding increases the consumption of type-lT consumers but not type-lO and type-hO consumers. Hence, the marginal value of real balances is given by (4.4). Finally, if $i \leq i_3$, the cost of holding real balances is sufficiently low that type-lO and type-hO consumers do not sell assets in the AM or use assets to purchase goods in the DM.

Next, we show the comparative statics of equilibrium outcomes with respect to interest rate (i) and the proportion of transparent meetings in the AM (τ). They include consumers' holdings of real balances (z), their consumption in the DM (q_p, which is by consumers in opaque meetings, and q^{hT} which is by consumers with high-quality assets in transparent meetings), total assets sold in the DM and used as payment in the DM ($s_p + \hat{a}_p$ and $s^{hT} + \hat{a}^{hT}$), and aggregate welfare, which is given by

$$W = \lambda\{(1-\rho)\tau[u(\hat{q}^{lT}) - \hat{q}^{lT}] + \rho\tau[u(\hat{q}^{hT}) - \hat{q}^{hT}] + (1-\tau)[u(\hat{q}_p) - \hat{q}_p]\}. \tag{4.6}$$

Note that the consumption by consumers with low-quality assets in transparent meetings is simply equal to z. In addition, s_p or $\hat{a}_p$ cannot be pinned down individually, since selling any positive amount of assets in the AM is a sufficient signal to producers in the DM that a consumer is a type-lO or type-hO consumer. Similarly, s^{hT} or $\hat{a}^{hT}$ cannot be pinned down individually. However, the sum of the assets sold in the AM and the assets used as payment in the DM can be determined.

Proposition 4 *Suppose $\rho\delta_h a < \tilde{q}$. Then, for any a, there exist $i_1 > i_2 \geq i_3 > 0$ such that the comparative statics of z, q_p, q^{hT}, $s_p + \hat{a}_p$, $s^{hT} + \hat{a}^{hT}$, and W with respect to increases in i and τ is given by the following table.*

Cases	z	q_p	q^{hT}	$s_p + \hat{a}_p$	$s^{hT} + \hat{a}^{hT}$	W
$i \geq i_1$	$\downarrow$	$\downarrow$	$-$	$-$	$\uparrow^{\dagger}$	$\downarrow$
$i_2 \leq i < i_1$	$-$	$-$	$-$	$-$	$-$	$-$
$i_3 \leq i < i_2$	$\downarrow$	$-$	$\downarrow^{*}$	$\uparrow$	$\uparrow^{\dagger}$	$\downarrow$
$i \leq i_3$	$\downarrow$	$\downarrow$	$\downarrow^{*}$	$-$	$\uparrow^{\dagger}$	$\downarrow$

(a) Comparative statics: an increase in i

Cases	z	q_p	q^{hT}	$s_p + \hat{a}_p$	$s^{hT} + \hat{a}^{hT}$	W
$i \geq i_1$	$\uparrow$	$\uparrow$	$\uparrow^{*}$	$-$	$\downarrow^{\dagger}$	$\uparrow$
$i_2 \leq i < i_1$	$-$	$-$	$-$	$-$	$-$	$\downarrow$
$i_3 \leq i < i_2$	$\uparrow$	$-$	$\uparrow^{*}$	$\downarrow$	$\downarrow^{\dagger}$	$\uparrow$
$i \leq i_3$	$\downarrow$	$\downarrow$	$\downarrow^{*}$	$-$	$\uparrow^{\dagger}$	$\downarrow$

(b) Comparative statics: an increase in τ

Note: $\uparrow$ means"increase"; $\downarrow$ means "decrease"; $-$ means "no change"
*: no change if $z + \delta_h a \geq q^*$; †: no change if $s^{hT} + \hat{a}^{hT} = a$

Table 1: Comparative Statics: $\rho\delta_h a < \tilde{q}$

Next, suppose $\rho\delta_h a \geq \tilde{q}$. Then, for any a, there exists $i^{\dagger}$ such that such that the comparative statics is given by the following table.

Cases	z	q_p	q^{hT}	$s_p + \hat{a}_p$	$s^{hT} + \hat{a}^{hT}$	W
$i > i^{\dagger}$	$\downarrow$	$-$	$\downarrow^{*}$	$\uparrow$	$\uparrow^{\dagger}$	$\downarrow$
$i \leq i^{\dagger}$	$\downarrow$	$\downarrow$	$\downarrow^{*}$	$-$	$\uparrow^{\dagger}$	$\downarrow$

(a) Comparative statics: an increase in i

Cases	z	q_p	q^{hT}	$s_p + \hat{a}_p$	$s^{hT} + \hat{a}^{hT}$	W
$i > i^{\dagger}$	$\uparrow$	$-$	$\uparrow^{*}$	$\downarrow$	$\downarrow^{\dagger}$	$\uparrow$
$i \leq i^{\dagger}$	$\downarrow$	$\downarrow$	$\downarrow^{*}$	$-$	$\uparrow^{\dagger}$	$\downarrow$

(b) Comparative statics: an increase in τ

Note: $\uparrow$ means"increase"; $\downarrow$ means "decrease"; $-$ means "no change"
*: no change if $z + \delta_h a \geq q^*$; †: no change if $s^{hT} + \hat{a}^{hT} = a$

Table 2: Comparative Statics: $\rho\delta_h a \geq \tilde{q}$

Proof: See Appendix B.

In what follows, we use numerical examples to help explain the comparative statics with respect to i and τ. First, we fix τ and consider the effect of i. In this example, $\rho\delta_h a < \tilde{q}$.[7] Under the parameters chosen, we have $i_2 > i_3$ so there are three cutoff values of i. Figure 1 shows how the consumption by type-lO and type-hO consumers, q_p, and the consumption by type-hT consumers, q^{hT}, depend on i. Note that the consumption by type-lT consumers is equal to z. When $i \geq i_1$, both q_p and q^{hT} are constrained by z. Since when $i_2 \leq i < i_1$, the choice of z does not depend on i.

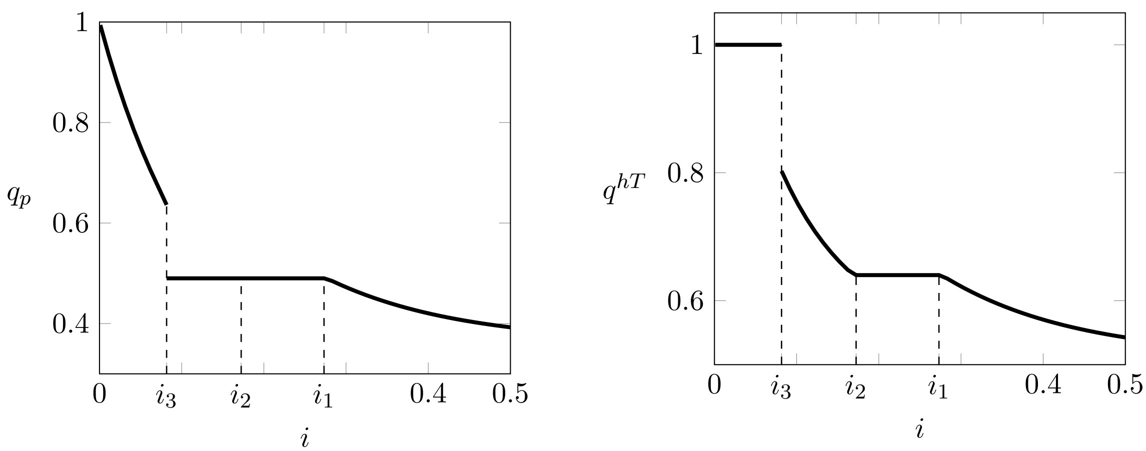


Figure 1: Equilibrium consumption and i

When $i_3 < i < i_2$, a larger z increases q^{hT} but $q_p = \tilde{q}$. In this case, type-lO and type-hO consumers simply reduce the amount of assets that they sell in the AM or use as payment in the DM while increasing the payment made in money in the DM. This allows type-hO to lower the loss from selling assets at a discount. Type-lO consumers must copy their strategies so that they are not identified as having low-quality assets. Finally, if $i \leq i_3$, q_p is increasing in i, because type-lO and type-hO consumers do not sell assets in the AM, and they use only money in the DM to purchase the DM good. In this particular example, type-q^{hT} consumers' portfolio of money and assets allow them to consumer the efficient amount when $i \leq i_3$.

Figure 2 shows how the sum of the assets sold in the AM (s_p and s^{hT}) and the assets used as payment in the DM ($\hat{a}_p$ and $\hat{a}^{hT}$) depend on i. Note that when $i \geq i_1$, both type-lO and type-hO consumers and type-hT consumers are constrained by their assets. Type-hT consumers are also constrained by their assets when $i_3 \leq i < i_1$, but type-lO and type-hO consumers are not. Specifically, when $i_3 \leq i < i_1$, type-lO and type-hO consumers will not consume more even if they have more assets. This is because, for type-hO consumers, the marginal benefit of consumption is lower than the marginal cost, which is selling assets at a discounted price, for any $q_p \geq \tilde{q}$.

[7] Other parameter values chosen are: $u(q) = \frac{q^{1-\sigma}}{1-\sigma}$, $\sigma = 0.5$, $\lambda = 0.5$, $\rho = 0.7$, $\tau = 0.5$, $a = 0.5$, and $\delta_h = 1$.

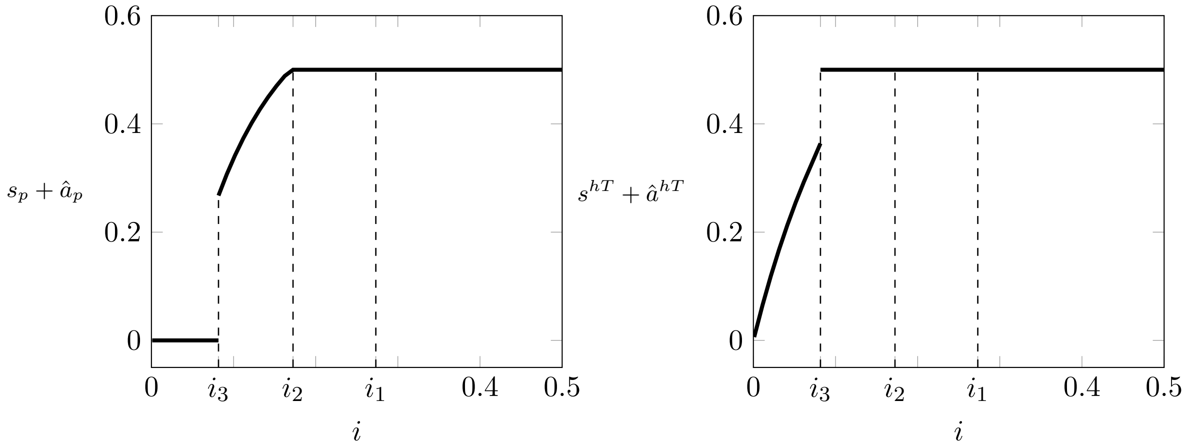


Figure 2: Assets sold in the AM (s_p and s^{hT}) and used as payment in the DM ($\hat{a}_p$ and $\hat{a}^{hT}$)

The right panel of Figure 3 shows how i affects aggregate welfare. Production and consumption in the CM good does not appear in (4.6) because they sum to zero.

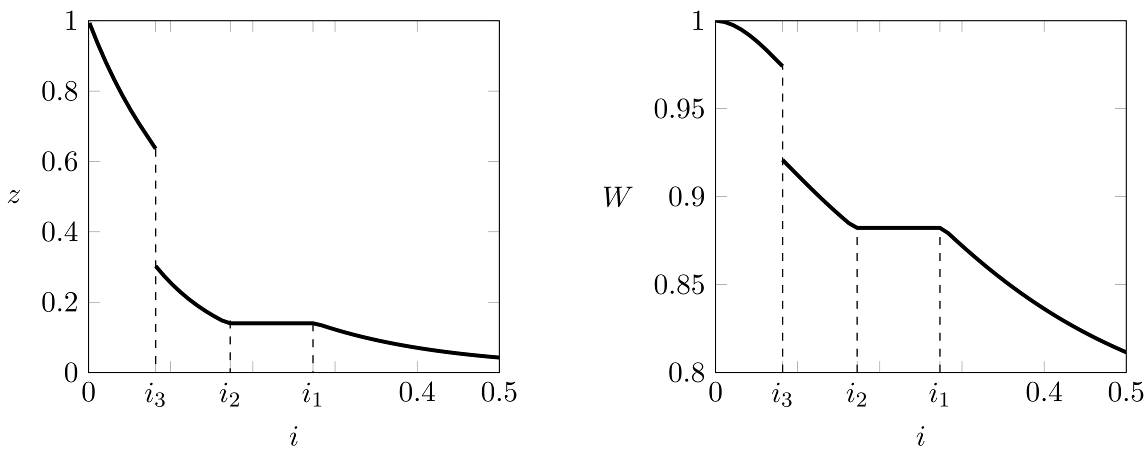


Figure 3: Equilibrium choice of real balances and equilibrium welfare

Aggregate welfare exhibits a drop at $i = i_3$ because agents' holdings of more real balances decrease discontinuously at $i = i_3$. Recall that when $i_3 \leq i < i_2$, $q_p = \tilde{q}$ and is independent of z, so the marginal value of real balances is low. Once $z \geq \tilde{q}$, $q_p = z$ and the marginal value of real balances jumps up. This leads agents to hold more real balances and consume more in the DM. Intuitively, the information friction in the AM and DM means that type-hO consumers are unwilling to consume more than $\tilde{q}$ unless they have enough real balances to avoid using assets at all. This means that type-hO consumers' consumption is also low when i is high since they always mimic the strategies of type-hO consumers.

Next, we show how τ, the share of transparent meetings in the AM, affects equilibrium outcomes. We refer to the four scenarios in Proposition 3 as Case 1 to 4. As shown by Figure 4 and Figure 5, changing τ leads the equilibrium to switch from Case 2 to Case 4.[8]

[8]When i is high, it is also possible that the equilibrium switches from Case 2 to Case 3, or from Case 1 to

In Case 2, $q_p = \tilde{q}$ and $q_h = \tilde{q} + (1-\rho)\delta_h a$. Hence, they are unaffected by the value of τ. Once the equilibrium switches to Case 4, however, an increase in τ lowers agents' incentive to carry money. This is because type-hT consumers can sell assets in the AM or use assets directly in the DM, and a higher τ makes it more likely that a consumer will become type-hT. Hence, we see a drop in the real balances agents hold as τ increases (see the left panel Figure 5). Because all consumers other than type-hT consumers consume less in the DM, aggregate welfare decreases in τ.

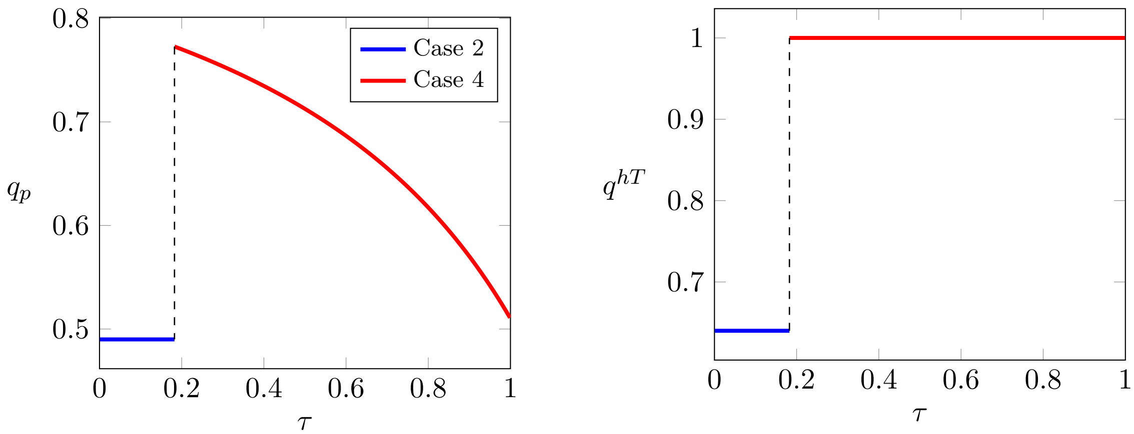


Figure 4: Equilibrium consumption and τ ($i = 0.06$)

Notice that aggregate welfare also decreases in τ when the equilibrium is in Case 2. This is because a higher τ means a larger share of consumers will become type-lT and will not be able to use assets at all in the AM or DM. This lowers the consumption in the DM. In other words, the existence of opaque meetings in this case is beneficial for aggregate welfare, because it allows some consumers with low-quality assets, i.e. type-lO consumers, to utilize their assets either to obtain money in the AM or to purchase goods in the DM.

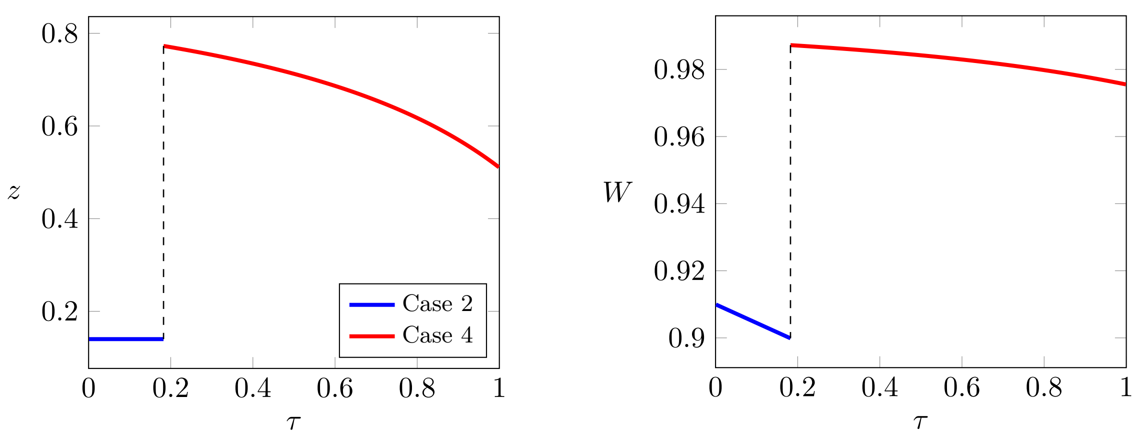


Figure 5: Equilibrium choice of real balances and equilibrium welfare ($i = 0.06$)

Case 2, and then to Case 3. Examples of these scenarios can be found in Appendix D.

This finding is similar to that in Andolfatto, Berentsen, and Waller (2014), where disclosing the information about asset quality can lead to assets losing their function as payment instruments. This in turn prevents trade that relies on such payment instruments from happening, thereby hurting welfare. However, it should be emphasized that although welfare is also decreasing in τ in Case 4, it is because consumers carry less real balances.

4.2 The case where $\delta_l > 0$

In this section, we consider the case where $\delta_l > 0$. Compared to when $\delta_l = 0$, consumers with low-quality assets can obtain money from the AM even if they are in transparent meetings. Furthermore, in both the AM and the DM, it is possible to have separating equilibria where consumers with low-quality and high-quality assets make different offers. Nevertheless, we show that the spirit of the main result remains unchanged. That is, consumers benefit from selling assets for money in the AM, even though assets have direct liquidity. This result holds even in opaque AM meetings, where the asymmetry of information is as severe as in the DM.

First, we present a useful observation that simplifies the analysis: although C-type consumers can in principle make separating offers in the AM, only pooling offers can be part of an equilibrium.

Lemma 1 *In equilibrium, C-type consumers only pool in opaque meetings.*

If C-types make separating offers in the AM, they will enter the DM with different asset portfolios depending on the quality of their assets. This means that producers can identify the C-types with high-quality assets through their asset portfolios, which in turn suggests that in the AM, C-types with low-quality assets will always mimic the strategies of those with high-quality assets. Hence, C-types do not make separating offers in the AM.

We summarize undefeated equilibria in the AM and the DM given consumers' portfolios (z_s, z_b, a) in the following proposition.

Proposition 5 *Assume that* $(1-\rho)\delta_l + (2\rho-1)\delta_h > 0$. *Conditional on* (z_s, z_b, a), *undefeated equilibria in the AM and the DM satisfy the following conditions.*

(1) $(\psi^{hT}, s^{hT}; \hat{q}^{hT}, \hat{z}^{hT}, \hat{a}^{hT})$ *is given by*

(a) $\psi^{hT} = \delta_h$; (b) $s^{hT} = (0, \min\{z_b/\delta_h, a\}]$;

(c) $\hat{q}^{hT} = \min\{q^*, z_s + \delta_h a\}$; (d) $\hat{z}^{hT} = z_s + \psi^{hT} s^{hT}$; (e) $\hat{a}^{hT} = \frac{\hat{q}^{hT} - \hat{z}^{hT}}{\delta_h}$.

(2) $(\psi^{lT}, s^{lT}; \hat{q}^{lT}, \hat{z}^{lT}, \hat{a}^{lT})$ *is given by*

(a) $\psi^{lT} = \delta_l$; (b) $s^{lT} = [0, \min\{z_b/\delta_l, a\}]$;

(c) $\hat{q}^{lT} = \min\{q^*, z_s + \delta_l a\}$; (d) $\hat{z}^{lT} = z_s + \psi^{lT} s^{lT}$; (e) $\hat{a}^{lT} = \frac{\hat{q}^{lT} - \hat{z}^{lT}}{\delta_l}$.

(3) *Define* $q_l(s) = \min\{z_s + \bar{\delta}s + \delta_l(a-s), q^*\}$ *and* $q_h(s) = z_s + \bar{\delta}s + \delta_h\hat{a}(s)$*, where* $\hat{a}(s)$ *solves*

$$u(q_l(s)) - q_l(s) = u(q_h(s)) - z_s - \bar{\delta}s - \delta_l\hat{a}. \tag{4.7}$$

Let $s^\dagger$ *solve*

$$\frac{[(1-\rho)\delta_l + (2\rho-1)\delta_h]u'(q_h(s^\dagger))}{\rho\delta_h} + [u'(q_l(s^\dagger)) - 1][u'(q_h(s^\dagger)) - 1] = 1. \tag{4.8}$$

There exist $0 \le z_s' < z_s''$ *such that if* $z_s \ge z_s''$*,*

(a) $s^{lO} = s^{hO} = s_p = 0$; (b) $\hat{q}^{lO} = q_l(s_p)$ *and* $\hat{q}^{hO} = q_h(s_p)$;
(c) $\hat{z}^{lO} = \hat{z}^{hO} = z_s$; (d) $\hat{a}^{lO} = \frac{\hat{q}^{lO} - \hat{z}^{lO}}{\delta_l}$ *and* $\hat{a}^{hO} = \hat{a}(0)$.

If $z_s' \le z_s < z_s''$*,*

(a) $\psi^{lO} = \psi^{hO} = \psi_p = (1-\rho)\delta_l + \rho\delta_h$; (b) $s^{lO} = s^{hO} = s_p = \min\{s^\dagger, z_b\}$;
(c) $\hat{q}^{lO} = q_l(s_p)$ *and* $\hat{q}^{hO} = q_h(s_p)$; (d) $\hat{z}^{lO} = \hat{z}^{hO} = z_s + \psi_p s_p$;
(e) $\hat{a}^{lO} = \frac{\hat{q}^{lO} - \hat{z}^{lO}}{\delta_l}$ *and* $\hat{a}^{hO} = \hat{a}(s_p)$.

If $z_s < z_s'$*,*

(a) $\psi^{lO} = \psi^{hO} = \psi_p = (1-\rho)\delta_l + \rho\delta_h$;
(b) $s^{lO} = s^{hO} = s_p = (0, \min\{z_b/[(1-\rho)\delta_l + \rho\delta_h], a\}]$;
(c) $\hat{q}^{lO} = \hat{q}^{hO} = z_s + \psi_p a$; (d) $\hat{z}^{lO} = \hat{z}^{hO} = z_s + \psi_p s_p$; (e) $\hat{a}^{lO} = \hat{a}^{hO} = a - s_p$.

Proof: See Appendix B.

Undefeated equilibria in transparent meetings are similar to the case where $\delta_l = 0$, with the only difference being that type-lO consumers may also sell their assets. However, because type-hT, type-lO and type-hO consumers can differentiate themselves from type-lT consumers through selling assets in the AM, type-lT consumers may only trade their assets at a price equal to δ_l in both the AM and the DM.

Compared to the case where $\delta_l = 0$, the equilibria in opaque meetings are different, and they depend on C-type consumers' money holding, z_s. When z_s is large (i.e., $z_s \ge z_s''$), C-type consumers do not sell in the AM, and they make separating offers in the DM. However, when z_s is small (i.e., $z_s < z_s''$), separating in the DM is costly for the C-types with high-quality assets, because they need to ration the use of assets as payment in order to signal asset quality to producers. This gives such consumers the incentive to sell some of their assets in the AM provided that the price discount in the AM is not too severe.[9] Such a strategy allows them to bring more real balances to the DM and rely less on the real asset. As a result, C-types with high-quality assets achieve both higher consumption (when compared to only separating

[9]This explains the assumption in Proposition 5 that $(1-\rho)\delta_l + (2\rho-1)\delta_h > 0$, which holds for a sufficiently large ρ.

in the DM) and higher average price for their assets (when compared to pooling in both the AM and the DM). This equilibrium in the AM and the DM, which combines the features of a separating equilibrium and a pooling equilibrium, is unique to our environment, because the real assets can be used to provide both indirect and direct liquidity. Finally, if $z < z_s'$, C-types will pool in the DM after selling assets in the AM, which is similar to the equilibria where $\delta_l = 0$. We show in the proof that the cutoff value, z_s', is strictly positive if and only if a is small. This is because when a is small, the amount of real balances C-types can obtain from the AM is low,. In such case, separating in the DM is costly for the C-types with high-quality assets if their money holding, z_s, is also small.

Despite the differences, the main insights from the cases where $\delta_l > 0$ and $\delta_l = 0$ are the same: consumers benefit from selling assets for money in the AM even though assets can provide direct liquidity, because such a strategy allows consumers to reduce/avoid the information asymmetry in the DM. More importantly, the result holds even if consumers are in opaque AM meetings, where the information asymmetry is as severe as in the DM.

Finally, we solve for the consumers' optimal choice of real balances in the CM. Similar to the case where $\delta_l = 0$, we focus on symmetric solutions, so $z_s = z_b = z$. The following proposition shows that how inflation affects C-type consumers' choice is in the AM and the DM, and it corresponds directly to the results in Proposition 5. The solution to optimal choice of real balances can also be found in the proof of the proposition.

Proposition 6 *Assume that $z_s' \geq \bar{\delta}a$. There exist μ' and μ'' such that*
(1) *If $\mu > \mu''$, consumers in opaque meetings sell assets in the AM and pool in the DM.*
(2) *If $\mu' < \mu \leq \mu''$, consumers in opaque meetings sell assets in the AM and separate in the DM.*
(3) *If $\mu \leq \mu'$, consumers in opaque meetings separate in the DM and do not trade in the AM.*
Proof: See Appendix B.

5 Endogenizing the Probability of Transparent Meetings (τ)

In this section, we endogenize τ, the share of transparent meetings in the AM. Specifically, a consumer may pay a utility cost κ and enter a transparent meeting in the AM, while asset quality remains to be private information in the DM. We consider two scenarios. In the first scenario, a consumer must make the choice of whether to pay κ after she learns her η_t (i.e., whether she is a C-type or N-type consumer) but before she learns the quality of her assets, δ. In the second scenario, a consumer make the choice after she learns η_t and δ. The results are similar for the cases where $\delta_l > 0$ and $\delta_l = 0$; to keep the analysis concise, here we focus on the case where $\delta_l = 0$ and relegate the other case to Appendix C.

5.1 κ paid before the quality shock

Given z, define the benefit of paying κ and entering a transparent meeting as $\mathcal{B}_1$.

$$\mathcal{B}_1(z) = (1-\rho)[u(\hat{q}^{lT}) + z - \hat{q}^{lT}] + \rho[u(\hat{q}^{hT}) + z + \delta_h a - \hat{q}^{hT}] - [u(\hat{q}_p) + z + \rho\delta_h a - \hat{q}_p], \tag{5.1}$$

where $\hat{q}^{lT} = \min\{z, q^*\}$, $\hat{q}^{hT} = \min\{z + \delta_h a, q^*\}$, and $\hat{q}_p = \max\{\min\{z, q^*\}, \min\{z + \rho\delta_h a, \tilde{q}\}\}$ where $u'(\tilde{q}) = 1/\rho$. To understand the expression, note that if a consumer pays κ, she will be in a transparent meeting regardless of the quality of the assets. Her surplus then follows from Proposition 2. If she does not pay the cost, she will be in an opaque meeting, and her expected value is $u(\hat{q}_p) + z + \rho\delta_h a - \hat{q}_p$, where $\hat{q}_p$ is again given by Proposition 2. The following lemma shows how $\mathcal{B}_1(z)$ depends on z.

Lemma 2 *Assume $u'''(.) > 0$ and $\delta_h a < q^*$. Then, $\mathcal{B}_1'(z) > 0$ for all $z \leq \tilde{q}$ and $\mathcal{B}_1'(z) < 0$ for all $\tilde{q} < z < q^*$. In addition, there exists $0 < \tilde{z} < \tilde{q}$ such that $\mathcal{B}_1(\tilde{z}) = 0$. Finally, $\mathcal{B}_1(q^*) = 0$.*
Proof: See Appendix B.

The proposition shows that, firstly, being in a transparent meeting is not always beneficial *ex ante*. This is because opaque meetings allow agents with low-quality assets to also use their assets in the AM and DM, thereby providing insurance against the quality shock. However, as agents' money holdings increase, they will be able to consume more even if they end up having low-quality assets. As a result, the insurance benefit decreases with z (i.e., $\mathcal{B}_1'(z) > 0$ for all $z \leq \tilde{q}$). If $z > \tilde{q}$, agents in opaque meetings do not sell assets in the AM or use assets directly in the DM. Hence, they consume less compared to agents with high-quality assets. However, as z increases, the difference in consumption becomes smaller. Hence, $\mathcal{B}_1'(z) < 0$ for all $z > \tilde{q}$, and $\mathcal{B}_1(q^*) = 0$.

From the proposition, we can conclude that $\mathcal{B}_1(z)$ reaches its maximum at $z = \tilde{q}$. Therefore, if $\kappa \geq \mathcal{B}_1(\tilde{q})$, no consumers will pay to be in transparent meetings. We assume agents do not pay the cost if they are indifferent. The following proposition summarizes the equilibrium when $\kappa < \mathcal{B}_1(\tilde{q})$.

Proposition 7 *Assume that $0 < \kappa < \mathcal{B}_1(\tilde{q})$, $u'''(.) > 0$, and $a < \frac{\tilde{q}}{\rho\delta_h}$. Then, there exist $i_1 > i_2 \geq i_3 > 0$ such that*
(1) *for all $i \geq i_1$, z solves $i = \lambda[u'(z + \rho\delta_h a) - 1]$, and agents do not pay κ;*
(2) *for all $i_2 \leq i < i_1$, $z = \tilde{q} - \rho\delta_h a$, and agents do not pay κ;*
(3) *for all $i_3 < i < i_2$, z solves $i = \lambda\{(1-\rho)[u'(z) - 1] + \rho[u'(\min\{z + \delta_h a, q^*\}) - 1]\}$, and agents pay κ;*
(4) *for all $i \leq i_3$, z solves $i = \lambda[u'(z) - 1]$, and agents do not pay κ.*

Proof: See Appendix B.

The following proposition summarizes the comparative statics of equilibrium outcomes with respect to interest rate (i). They include consumers' holdings of real balances (z), consumption in the DM by consumers with high-quality assets (q_h), total assets sold in the DM and used as payment in the DM ($s + \hat{a}$), and aggregate welfare. Note that the consumption by consumers with low-quality assets is either equal to q^h if consumers do not pay κ, or z if consumers pay κ. In addition, if consumers do not pay κ, s or $\hat{a}$ cannot be pinned down individually, since consumers will be indifferent between selling the real asset in the AM and using them directly as payment in the DM. However, the sum of the assets sold in the AM and the assets used as payment in the DM can be determined. Finally, since κ is a fixed cost, within each of the four equilibrium cases (see Proposition 7), a change in κ does not affect z, q_h, or $s + \hat{a}$. Hence, we only consider the comparative statics with respect to i.

Proposition 8 *There exist $i_1 > i_2 \geq i_3 > 0$ such that such that the comparative statics of z, q_h, $s + \hat{a}$, and W with respect to an increase in i is given by the following table.*

Cases	z	q_h	$s + \hat{a}$	W
$i \geq i_1$	$\downarrow$	$-$	$-$	$\downarrow$
$i_2 \leq i < i_1$	$-$	$-$	$-$	$-$
$i_3 \leq i < i_2$	$\downarrow$	$\downarrow^*$	$\uparrow^\dagger$	$\downarrow$
$i \leq i_3$	$\downarrow$	$\downarrow$	$-$	$\downarrow$

Note: $\uparrow$ means"increase"; $\downarrow$ means "decrease"; $-$ means "no change"

*: no change if $z + \delta_h a \geq q^*$; †: no change if $s + \hat{a} = a$

Table 3: Comparative Statics: κ Paid Before the Quality Shock

Proof: See Appendix B.

We use the following numerical example to illustrate the results.[10] Similar to the scenario where τ is exogenous (see Proposition 3), there are either two or three cutoff values of i. When i is either too high or too low, consumers choose not to pay κ, and therefore they are in opaque meetings in the AM. In these cases, consumers with high-quality assets consume the same amount (q_h) as consumers with low-quality assets. When $i \leq i_3$, consumers do not sell assets in the AM or use assets as a medium of exchange in the DM. This is because the information asymmetry means that for consumers with high-quality assets, the marginal cost of selling or using assets (i.e., sacrificing the dividend) exceeds the marginal benefit of selling or using assets (i.e., more consumption).

[10]In this example, $u(q) = \frac{q^{1-\sigma}}{1-\sigma}$, $\sigma = 0.5$, $\lambda = 0.5$, $\rho = 0.7$, $\kappa = 0.004$, $a = 0.5$, and $\delta_h = 1$.

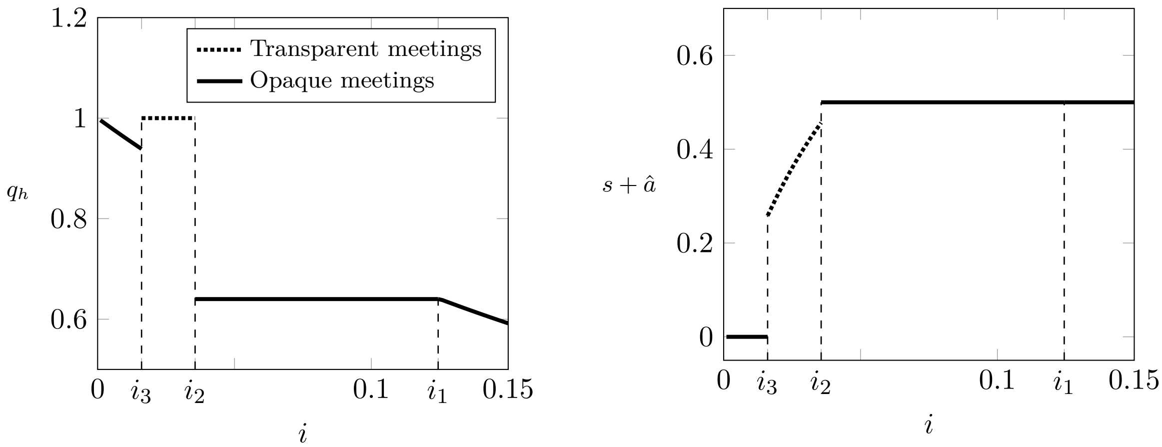


Figure 6: Consumption (q_h) and assets sold or used as payment ($s + \hat{a}$)

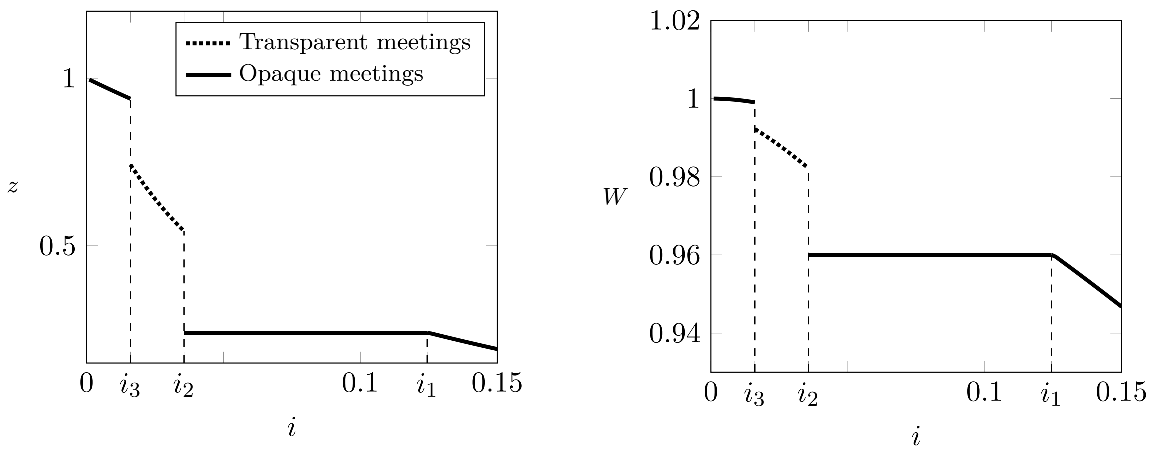


Figure 7: Real balances (z) and aggregate welfare (W)

When $i_2 \leq i < i_1$, however, the marginal cost of selling or using assets is equal to the marginal benefit of selling or using assets for consumers with high-quality assets. In this scenario, having more real balances simply means that consumers substitute assets for real balances. Since holding real balances is costly for any $i > 0$, agents only hold enough real balances so that their assets portfolios allow them to consume $\tilde{q}$. Recall that s or $\hat{a}$ cannot be pinned down individually. This is because when consumers have transparent meetings, selling any positive amount of assets in the AM is a sufficient signal to producers in the DM that a consumer has high-quality assets. If instead consumers have opaque meetings, they are indifferent between selling assets in the AM or using them directly in the DM, since the information asymmetry is the same in the AM and the DM. However, in both cases, the sum of the assets sold in the AM and the assets used as payment in the DM can be determined.

We also check how the equilibrium depends on κ for a given i. Unsurprisingly, agents prefer opaque meetings in the AM when κ is sufficiently large. Since κ is a fixed cost, it does not otherwise affect equilibrium outcomes.

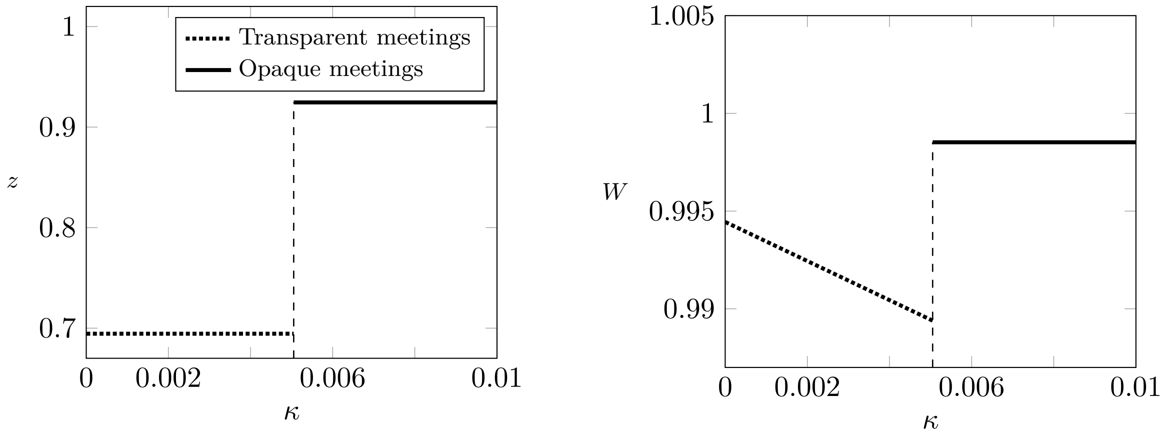


Figure 8: Real balances (z) and aggregate welfare (W) ($i = 0.02$)

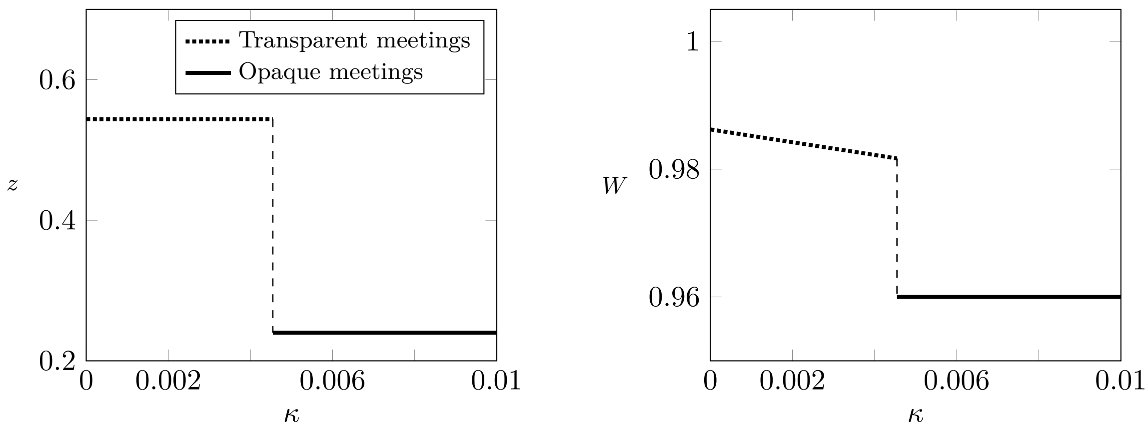


Figure 9: Real balances (z) and aggregate welfare (W) ($i = 0.04$)

Interestingly, aggregate welfare may increase discontinuously when consumers switch to not paying κ. This is because, in this particular example, i is low, so if consumers do not pay κ (i.e., the black lines in Figure 8), they also do not sell or use assets (see case (4) in Proposition 7). Therefore, consumers hold more real balances relative to when κ is low. This increases the consumption of consumers with low-quality assets relative to when consumers have transparent meetings in the AM. Consequently, aggregate welfare is higher. However, the opposite happens, when inflation is high: consumers benefit more from pooling in the AM, which allows them to use the real assets to either obtain money or directly as MoE.

5.2 κ paid after the quality shock

Next, we consider the second scenario, where consumers choose whether to pay κ after learning the quality of their assets. Given (z, a), define the benefit of paying κ and entering a transparent meeting for a consumer with high-quality assets as $\mathcal{B}_2$. Note that consumers with low-quality assets do not have the incentive to pay κ, since they do not benefit from disclosing the quality of their assets. Let $\tilde{\tau}$ denote the fraction of consumers who choose to pay κ.

$$\mathcal{B}_2(z,\tilde{\tau}) = u(\hat{q}^{hT}) + z + \delta_h a - \hat{q}^{hT} - [u(\hat{q}(\tilde{\tau})) + z + \rho\delta_h a - \hat{q}(\tilde{\tau})], \tag{5.2}$$

where $\hat{q}^{hT} = \min\{z+\delta_h a, q^*\}$, and $\hat{q}(\tilde{\tau}) = \max\{z, \min\{z+\xi(\tilde{\tau})\delta_h a, q^\dagger(\tilde{\tau})\}\}$ where $u'(q^\dagger(\tilde{\tau})) = 1/\xi(\tilde{\tau})$, and $\xi(\tilde{\tau}) = \frac{\rho(1-\tilde{\tau})}{\rho(1-\tilde{\tau})+1-\rho}$ is the average asset quality among consumers who do not pay κ. Notice that now the benefit of having transparent meetings also depends on $\tilde{\tau}$.

Lemma 3 *$\mathcal{B}_2(z,\tilde{\tau})$ is decreasing in z for any $z < q^\dagger(\tilde{\tau}) - \xi(\tilde{\tau})\delta_h a$ and $z > q^\dagger(\tilde{\tau})$ and increasing in z for any $z \in [q^\dagger(\tilde{\tau}) - \xi(\tilde{\tau})\delta_h a, q^\dagger(\tilde{\tau})]$, while $\mathcal{B}_2(z,\tilde{\tau})$ is increasing in $\tilde{\tau}$.*
Proof: See Appendix B.

Now, define $\bar{\mathcal{B}}_2(z)$ to be the upper bound of the benefit of paying κ, which happens when $\tilde{\tau} = 1$.

$$\bar{\mathcal{B}}_2(z) = u(\hat{q}^{hT}) + z + \delta_h a - \hat{q}^{hT} - [u(z) + z + \rho\delta_h a - z]. \tag{5.3}$$

Next, define $\underline{\mathcal{B}}_2(z)$ to be the lower bound of the benefit of paying κ, which happens when $\tilde{\tau} = 0$.

$$\underline{\mathcal{B}}_2(z) = u(\hat{q}^{hT}) + z + \delta_h a - \hat{q}^{hT} - [u(\hat{q}(0)) + z + \rho\delta_h a - \hat{q}(0)]. \tag{5.4}$$

If $\kappa \geq \bar{\mathcal{B}}_2(z)$, then no agents will pay κ. If $\kappa \leq \underline{\mathcal{B}}_2(z)$, all agents will pay κ. If $\kappa \in (\underline{\mathcal{B}}_2(z), \bar{\mathcal{B}}_2(z))$, then there exists $\tilde{\tau}'(z)$ such that for all $\tilde{\tau}' \geq \tilde{\tau}(z)$, agents will pay κ, and $\tilde{\tau}'$ solves

$$u(\hat{q}^{hT}) + z + \delta_h a - \hat{q}^{hT} - [u(\hat{q}(\tilde{\tau}'(z))) + z + \rho\delta_h a - \hat{q}(\tilde{\tau}'(z))] = \kappa. \tag{5.5}$$

Now, we solve for the optimal choice of z in the CM. The following lemma simplifies the analysis.

Lemma 4 *If z is chosen optimally in the CM, then either $\tilde{\tau} = 1$ or $\tilde{\tau} = 0$.*
Proof: See Appendix B.

The lemma shows that as long as z is chosen optimally, consumers do not play mixed strategies in the AM when deciding whether to pay κ. Intuitively, this is because the marginal value of real balances depends on whether κ is paid. Therefore, even if consumers are indifferent ex post regarding paying κ, they will strictly prefer different holdings of real balances ex ante. Now, let $\iota(\delta, z)$ denote the choice of whether to pay κ or not conditional on the realization of δ and z. The following proposition shows that except at one cutoff value (i_2'), there is a unique combination of z and $\iota(z,\delta)$ that maximizes consumers' expected utility.

Proposition 9 *Assume that* $0 < \kappa < \bar{\mathcal{B}}_2(0)$, $u'''(.) > 0$, *and* $a < \frac{\tilde{q}}{\rho\delta_h}$. *Then, there exist* $i'_1 \geq i'_2 \geq i'_3 \geq i'_4 > 0$ *such that*
(1) *for all* $i \geq i'_1$, z *solves* $i = \lambda\{(1-\rho)[u'(z)-1] + \rho[u'(\min\{q^*, z+\delta_h a\}) - 1]\}$, *and agents pay* κ *in the AM;*
(2) *for all* $i'_2 \leq i < i'_1$, $z = z_1$ *where* z_1 *solves* $i'_1 = \lambda\{(1-\rho)[u'(z)-1] + \rho[u'(\min\{q^*, z+\delta_h a\}) - 1]\}$, *and agents pay* κ *in the AM;*
(3) *for all* $i'_3 \leq i < i'_2$, z *solves* $i = \lambda[u'(z+\rho\delta_h a) - 1]$, *and agents do not pay* κ *in the AM.*
(4) *for all* $i'_4 \leq i < i'_3$, z *solves* $i'_3 = \lambda[u'(z+\rho\delta_h a) - 1]$, *and agents do not pay* κ *in the AM.*
(5) *for all* $i < i'_4$, z *solves* $i = \lambda[u'(z) - 1]$, *and agents do not pay* κ *in the AM.*
Proof: See Appendix B.

Recall that if κ is paid before consumers learn the quality of their assets, consumers do not pay κ when i is either too low or too high (see Proposition 7). However, if κ is instead paid after consumers learn asset quality, then consumers pay κ as long as i is high but does not pay when i is low. To understand the difference, note that in opaque meetings, consumers receive insurance against the quality shock, since those with low-quality assets are able to sell their assets. Such insurance is especially beneficial when i is high, and consumers' money holdings are small. This is why consumers do not pay κ if it has to be paid before they learn asset quality. However, if consumers pay κ after they learn asset quality, such insurance no longer exists. When i is high, and consumers' money holdings are small, paying κ and avoiding information asymmetry become beneficial for consumers with high-quality assets.

The following proposition summarizes the comparative statics of equilibrium outcomes with respect to interest rate (i).

Proposition 10 *There exist* $i'_1 \geq i'_2 \geq i'_3 \geq i'_4 > 0$ *such that such that the comparative statics of* z, q_h, $s+\hat{a}$, *and* W *with respect to an increase in* i *is given by the following table.*

Cases	z	q_h	$s+\hat{a}$	W
$i \geq i'_1$	$\downarrow$	$\downarrow^*$	$\uparrow^\dagger$	$\downarrow$
$i'_2 \leq i < i'_1$	$-$	$-$	$-$	$-$
$i'_3 \leq i < i'_2$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$
$i'_4 \leq i < i'_3$	$-$	$-$	$-$	$-$
$i < i'_4$	$\downarrow$	$\downarrow$	$-$	$\downarrow$

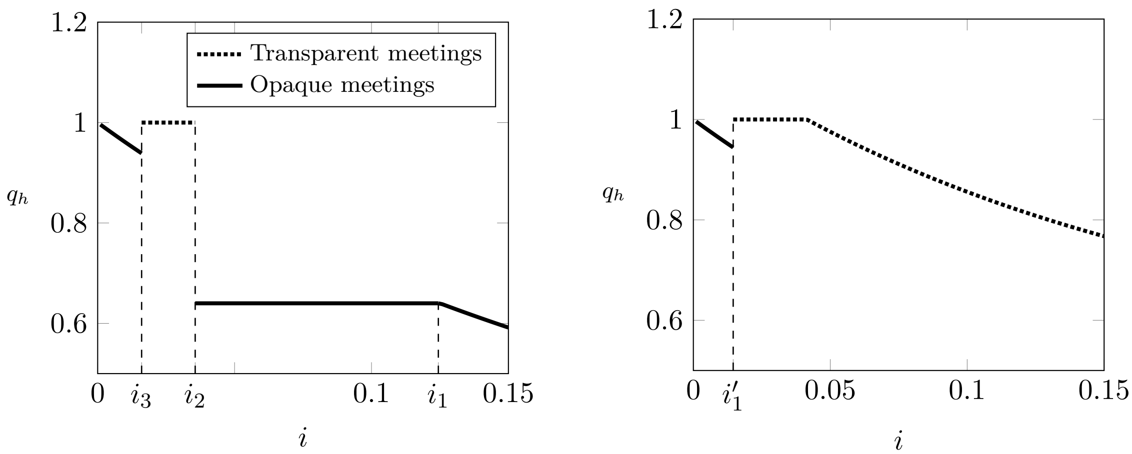
Note: $\uparrow$ means"increase"; $\downarrow$ means "decrease"; $-$ means "no change"
*: no change if $z+\delta_h a \geq q^*$; †: no change if $s+\hat{a} = a$

Table 4: Comparative Statics: κ After the Quality Shock

Proof: See Appendix B.

We again use the a numerical example to further illustrate the results. We use the same parameter values as in the example shown in Figure 6 and Figure 7. In Figure 10, 11, and 12, we compare how equilibrium outcomes depend on when κ is paid by consumers. We use solid black line and dotted black line to represent equilibrium outcomes when consumers pay and do not pay κ in the AM after they learn asset quality, respectively. In this particular example, $i_1' = i_2' = i_3' = i_4'$ so there is only one cutoff value. We also reproduce equilibrium outcomes when consumers have to pay κ before they learn asset quality. There are three cutoff values, i_1, i_2, and i_3 in that scenario (see Section 5.1).

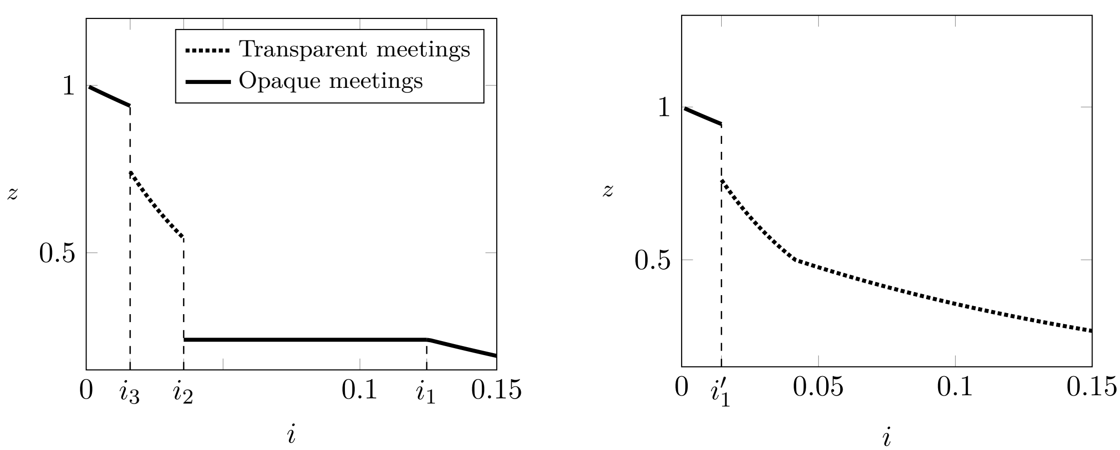
As shown in Proposition 9, the timing of the payment of κ makes a difference mainly when i is relatively large. When κ can be paid after the quality shock, consumers with high-quality assets pay it as long as $i > i_1'$, so the value of their assets is higher in the AM and DM. As a result, they consume more in the DM.



(a) κ paid before the quality shock

(b) κ paid after the quality shock

Figure 10: Consumption (q_h)

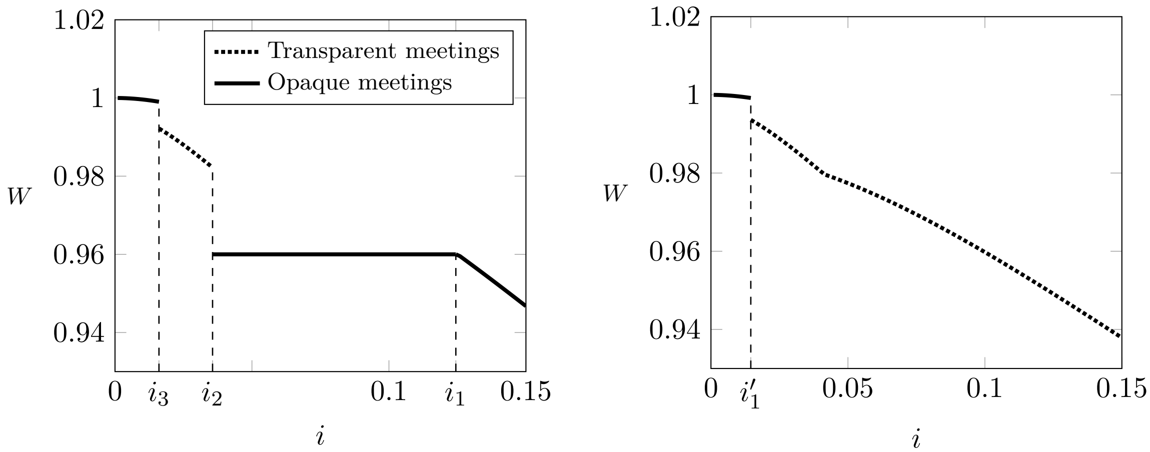


(a) κ paid before the quality shock

(b) κ paid after the quality shock

Figure 11: Real balances (z)

Note that because $q_l = z$ if $i > i_1'$ and κ is paid after the quality shock, consumers carry more money to insure against the quality shock if inflation is not too high. This explains why aggregate welfare, W, drops less significantly compared to to when κ has to be paid before the quality shock. However, if inflation is very high, the opposite happens: because compared to carrying real balances, the opaque meetings in the AM offers a better insurance against the quality shock. Consequently, welfare is higher if κ is paid before the quality shock.

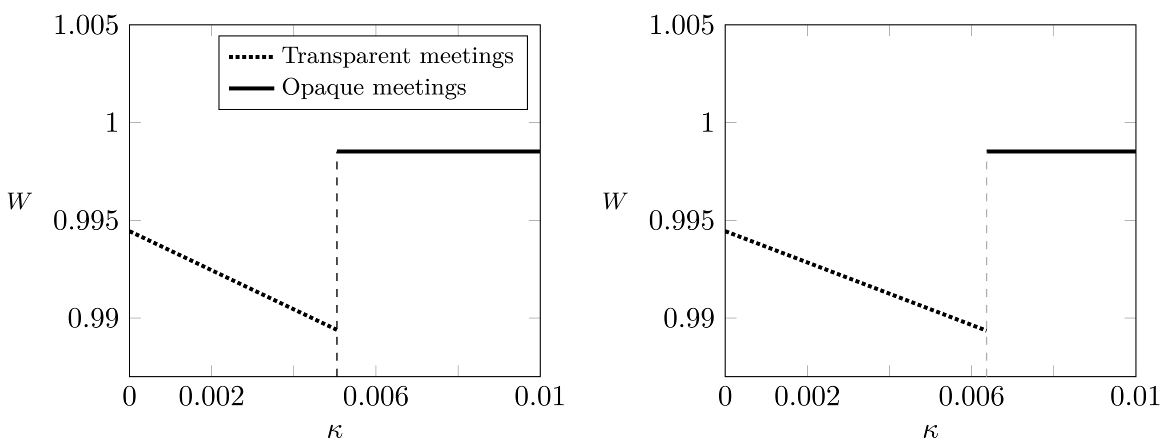


(a) κ paid before the quality shock

(b) κ paid after the quality shock

Figure 12: Aggregate welfare (W)

We also compare how equilibrium outcomes depend on κ in both scenarios. When inflation is low, the main difference is that the cutoff value at which consumers switch from paying κ to not paying κ increases when κ can be paid after the quality shock. This is because in that case, only consumers with high-quality assets pay the cost, which also explains why aggregate welfare is higher in Figure 13.

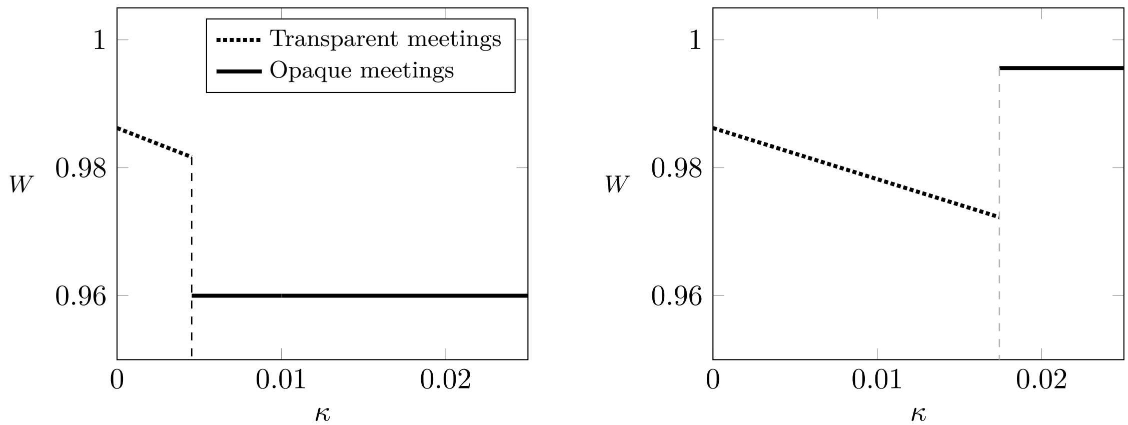


(a) κ paid before the quality shock

(b) κ paid after the quality shock

Figure 13: Aggregate welfare (W) ($i = 0.02$)

When inflation is high, however, welfare is monotonically decreasing in κ when it is paid before the quality shock. This is because when inflation is high, consumers benefit more from pooling in the AM, which allows them to use the real assets to either obtain money or directly as MoE. However, when inflation is low and κ is high, consumers hold more real balances. This increases the consumption of consumers with low-quality assets. Consequently, aggregate welfare increases discontinuously when consumers switch to opaque meetings.



(a) κ paid before the quality shock

(b) κ paid after the quality shock

Figure 14: Aggregate welfare (W) ($i = 0.04$)

6 Conclusion

In this paper, we develop a model where a real asset is subject to asymmetric information and serves a double liquidity role: it can compete directly with money as a medium of exchange or it can be liquidated for money in an over-the-counter secondary market. Thus, our model allows us to study how the degree of asymmetric information in the secondary market affects asset liquidity and aggregate welfare.

We start with a version of the model where the degree of asymmetric information in the asset market is exogenous. We find that rather than using the asset directly as a medium of exchange, agents prefer to liquidate it for money in the asset market. Furthermore, we show that a decrease in severity of asymmetric information in the asset market can hurt welfare, and that high inflation can lead to a discontinuous decrease in aggregate welfare. We also endogenize the degree of information asymmetry in the asset market by allowing agents to invest in information. We find that the timing of the investment (compared to the revelation of the asset's quality) is important. If the investment is made after the quality of the asset has been realized, we find that asset sellers pay that cost if and only if inflation is high. However, if the investment is made before the quality of the asset is realized, asset sellers only pursue it when inflation is neither too high nor too low. Thus, allowing for the endogenous determination of the information asymmetry highlights new insights regarding the liquidity role of assets, as money is no longer the unique form of insurance against the quality shock.

References

Akerlof, G. A. (1970). The market for "lemons": Quality uncertainty and the market mechanism. *The Quarterly Journal of Economics 84*(3), 488–500.

Altermatt, L., K. Iwasaki, and R. Wright (2022). General equilibrium with multiple liquid assets. Mimeo.

Andolfatto, D., A. Berentsen, and C. Waller (2014). Optimal disclosure policy and undue diligence. *Journal of Economic Theory 149*, 128–152.

Bajaj, A. (2018). Undefeated equilibria of the shi-trejos-wright model under adverse selection. *Journal of Economic Theory 176*(7), 957–986.

Berentsen, A., G. Camera, and C. Waller (2007). Money, credit and banking. *Journal of Economic Theory 135*(1), 171–195.

Berentsen, A., S. Huber, and A. Marchesiani (2014). Degreasing the wheels of finance. *International economic review 55*(3), 735–763.

Berentsen, A. and C. Waller (2011). Outside versus inside bonds: a modigliani–miller type result for liquidity constrained economies. *Journal of Economic Theory 146*(5), 1852–1887.

Boel, P. and G. Camera (2006). Efficient monetary allocations and the illiquidity of bonds. *Journal of Monetary Economics 53*(7), 1693–1715.

Cai, Z. and F. Dong (2020). A model of secular migration from centralized to decentralized trade. *Economic Theory*, forthcoming. `https://doi.org/10.1007/s00199-020-01280-2`.

Camargo, B. and B. Lester (2014). Trading dynamics in decentralized markets with adverse selection. *Journal of Economic Theory 153*, 534–568.

Carapella, F. and S. Williamson (2015). Credit markets, limited commitment, and government debt. *The Review of Economic Studies 82*(3), 963–990.

Chari, V. V., A. Shourideh, and A. Zetlin-Jones (2014). Reputation and persistence of adverse selection in secondary loan markets. *American Economic Review 104*(12), 4027–70.

Chiu, J. and T. V. Koeppl (2016). Trading dynamics with adverse selection and search: market freeze, intervention and recovery. *The Review of Economic Studies 83*(3), 969–1000.

Cho, I.-K. and D. M. Kreps (1987). Signaling games and stable equilibria. *The Quarterly Journal of Economics 102*(2), 179–221.

Choi, M. (2018). Imperfect information transmission and adverse selection in asset markets. *Journal of Economic Theory 176*, 619–649.

Eisfeldt, A. L. (2004). Endogenous liquidity in asset markets. *The Journal of Finance 59*(1), 1–30.

Geromichalos, A. and L. Herrenbrueck (2016). Monetary policy, asset prices, and liquidity in over-the-counter markets. *Journal of Money, Credit and Banking 48*(1), 35–79.

Geromichalos, A. and L. Herrenbrueck (2022). The liquidity-augmented model of macroeconomic aggregates: A new monetarist dsge approach. *Review of Economic Dynamics*.

Geromichalos, A., L. Herrenbrueck, and K. Salyer (2016). A search-theoretic model of the term premium. *Theoretical Economics 11*(3), 897–935.

Geromichalos, A., K. M. Jung, S. Lee, and D. Carlos (2021). A model of endogenous direct and indirect asset liquidity. *European Economic Review 132*, 103627.

Golosov, M., G. Lorenzoni, and A. Tsyvinski (2014). Decentralized trading with private information. *Econometrica 82*(3), 1055–1091.

Gorton, G. and G. Ordonez (2014). Collateral crises. *American Economic Review 104*(2), 343–78.

Guerrieri, V. and R. Shimer (2014). Dynamic adverse selection: a theory of illiquidity, fire sales, and flight to quality. *American Economic Review 104*(7), 1875–1908.

Herrenbrueck, L. (2019). Frictional asset markets and the liquidity channel of monetary policy. *Journal of Economic Theory 181*, 82–120.

Herrenbrueck, L. and A. Geromichalos (2017). A tractable model of indirect asset liquidity. *Journal of Economic Theory 168*, 252–260.

Kocherlakota, N. R. (2003). Societal benefits of illiquid bonds. *Journal of Economic Theory 108*(2), 179–193.

Kurlat, P. (2013). Lemons markets and the transmission of aggregate shocks. *American Economic Review 103*(4), 1463–89.

Lagos, R. and R. Wright (2005). A unified framework for monetary theory and policy analysis. *Journal of Political Economy 113*(3), 463–484.

Lauermann, S. and A. Wolinsky (2016). Search with adverse selection. *Econometrica 84*(1), 243–315.

Leland, H. E. and D. H. Pyle (1977). Informational asymmetries, financial structure, and financial intermediation. *The Journal of Finance 32*(2), 371–387.

Lester, B., A. Postlewaite, and R. Wright (2012). Information, liquidity, asset prices, and monetary policy. *The Review of Economic Studies 79*(3), 1209–1238.

Li, Y., G. Rocheteau, and P.-O. Weill (2012). Liquidity and the threat of fraudulent assets. *Journal of Political Economy 120*(5), 815–846.

Lu, X. (2022). Liquidity, asset prices and information acquisition. Mimeo.

Madison, F. (2017). Asymmetric information in frictional markets for liquidity: collateralized credit vs asset sale. Mimeo.

Madison, F. (2019). Frictional asset reallocation under adverse selection. *Journal of Economic Dynamics and Control 100*, 115–130.

Mailath, G. J., M. Okuno-Fujiwara, and A. Postlewaite (1993). Belief-based refinements in signalling games. *Journal of Economic Theory 60*(2), 241–276.

Mattesini, F. and E. Nosal (2016). Liquidity and asset prices in a monetary model with otc asset markets. *Journal of Economic Theory 164*, 187–217.

Ozdenoren, E., K. Yuan, and S. Zhang (2019). Dynamic coordination with flexible security design. Mimeo.

Plantin, G. (2009). Learning by holding and liquidity. *The Review of Economic Studies 76*(1), 395–412.

Rocheteau, G. (2008). Money and competing assets under private information. Mimeo.

Rocheteau, G. (2011). Payments and liquidity under adverse selection. *Journal of Monetary Economics 58*(3), 191–205.

Trejos, A. (1997). Incentives to produce quality and the liquidity of money. *Economic Theory 9*(2), 355–365.

Trejos, A. (1999). Search, bargaining, money, and prices under private information. *International Economic Review 40*(3), 679–696.

Velde, F. R., W. E. Weber, and R. Wright (1999). A model of commodity money, with applications to gresham's law and the debasement puzzle. *Review of Economic Dynamics 2*(1), 291–323.

Wallace, N. (1998). A dictum for monetary theory. *Federal Reserve Bank of Minneapolis Quarterly Review 22*(1), 20–26.

Wang, Z. (2020). Liquidity and private information in asset markets: To signal or not to signal. *Journal of Economic Theory 190*, 105122.

Williamson, S. and R. Wright (1994). Barter and monetary exchange under private information. *American Economic Review 84*(1), 104–123.

Appendix A Undefeated Equilibrium Refinement

In the first part of this section, we use a simplified version of the model to illustrate the logic behind the (original) Undefeated Equilibrium refinement as defined by Mailath et al. (1993) and why we use it instead of another popular refinement method, the Intuitive Criterion (Cho and Kreps, 1987). In the second part, we show that the version of Undefeated Equilibrium refinement used in this paper (see Definition 2) captures the same logic behind the original version.

I. Undefeated Equilibrium Refinement

First, consider a simplified version of the model. Assume there is no AM (the asset market) between the CM and DM. Suppose also that $\delta_l > 0$ so that separating equilibria are possible. For simplicity, assume $\lambda = 1$ so that all consumers want to consume in the DM. All the other elements of the model environment remain unchanged. Now, let $j \in \{l, h\}$ denote the quality of consumer's asset. Define $\underline{v}^j$, $j \in \{l, h\}$ to be the lowest possible surplus j-type consumer can obtain

$$\underline{v}^j \equiv \max_{q,z,a}\{u(q) - z - \delta_j a\} \text{ s.t. } \delta_l a + z \geq q.$$

It is straightforward to show that the full set of perfect Bayesian equilibria (PBE) can be described as follows.

(1) $u(q^j) - z^j - \delta_j a^j \geq v^j$ for $j \in \{l, h\}$ (individual participation constraint).

(2) $[\gamma(q^j, z^j, a^j)\delta_h + (1 - \gamma(q^j, z^j, a^j))\delta_l]a^j + z^j \geq q^j$ for $j \in \{l, h\}$.

(3) $\gamma(q^j, z^j, a^j)$ is given by

$$\gamma(q^j, z^j, a^j) = \begin{cases} 1, \text{ if } u(q^j) - z^j - \delta_l a^j \leq \underline{v}^l \text{ and } u(q^j) - z^j - \delta_h a^j \geq \underline{v}^h; \\ \rho, \text{ if } u(q^j) - z^j - \delta_l a^j > \underline{v}^l \text{ and } u(q^j) - z^j - \delta_h a^j \geq \underline{v}^h; \\ 0, \text{ if } u(q^j) - z^j - \delta_l a^j \geq \underline{v}^l \text{ and } u(q^j) - z^j - \delta_h a^j < \underline{v}^h. \end{cases}$$

(4) Off-equilibrium path beliefs: $\gamma(q, z, a) = 0$ for all $(q, z, a) \notin \{(q^l, z^l, a^l), (q^h, z^h, a^h)\}$.

It is clear that there exist a continuum of equilibria. Hence, equilibrium refinement is necessary for the analysis to be productive. Mailath et al., 1993 introduce a refinement method that can be used for two-player signaling games such as this one. It is defined as follows: a PBE (PBE-1) is defeated by another PBE (PBE-2) if there exists $(\hat{q}, \hat{z}, \hat{a})$ that satisfies

(1) Requirement 1: There exists $K \subseteq \{l, h\}$ such that $(\hat{q}, \hat{z}, \hat{a})$ is played by type-K consumers in PBE-2 but not in PBE-1.

(2) Requirement 2: There exists $J \subseteq K$ such that J-type consumers obtain strictly higher surplus in PBE-2.

(3) Requirement 3: In PBE-1,

$$\gamma^a(\hat{\psi}, \hat{s}) \neq \frac{\rho \mathbb{1}(hO \in J)}{\rho \mathbb{1}(hO \in K) + (1 - \rho)\mathbb{1}(lO \in K)}.$$

In words, requirements 1 and 2 say that there exists a strategy in PBE-2 that is not played in PBE-1, and such a strategy provides strictly higher payoff for some consumers. Requirement 3 says that the reason why the strategy $(\hat{q}, \hat{z}, \hat{a})$ is not played in PBE-1 is that producers in PBE-1 have not anticipated it. However, because such a deviation is profitable for some consumers, producers should have expected it. In other words, producers' beliefs in PBE-1 are arguably "unreasonable". A PBE is undefeated if there does not exist another PBE that satisfies requirements 1-3.

In addition to Undefeated Equilibrium refinement, another popular refinement method used in similar environments is the Intuitive Criterion (Cho and Kreps, 1987). For example, Rocheteau (2011) shows that a proposed equilibrium fails the Intuitive Criterion if there exists an unsent offer such that

(1) Under the unsent offer, l-type consumers' payoff is lower regardless of the inference the producer draws from the unsent offer. As a result, the producer should believe that the offer comes from h-types.

(2) Conditional on the producer believing that the offer comes from h-type consumers, the offer provides strictly higher payoff to h-type consumers.

The main difference between Undefeated Equilibrium refinement and the Intuitive Criterion is that the latter replies on off-equilibrium messages as signals. As pointed out by Mailath et al. (1993), if the producer does believe that the unsent offer only comes from h-type consumers, then the original equilibrium will no longer exist, since h-type consumers will always deviate. Then, for l-type consumers, their payoff in the current equilibrium *should* become irrelevant − instead, they should consider their payoff in the *new* equilibrium given the seller's belief. Undefeated Equilibrium refinement takes care of such a concern by considering alternative PBE (as opposed to an off-equilibrium message) when deciding whether an equilibrium passes the refinement. In the context of this simplified model, a separating equilibrium that passes the Intuitive Criterion also satisfies the Undefeated Equilibrium refinement. However, the Undefeated Equilibrium refinement also allows pooling equilibria if it is Pareto-optimal (for more details, see Bajaj (2018)).

II. Modified Undefeated Equilibrium Refinement

Now, we show that the version of Undefeated Equilibrium refinement found in this paper (Definition 2) shares the same basic logic as the original version defined in Mailath et al. (1993). To see this, first note that there are two important features of the signaling game in this paper.

(1) Both asset buyers and producers must form beliefs regarding asset quality based on the offer they receive and the asset portfolio of the consumer they are matched with.

(2) The game appears to have "two stages", i.e., the interactions between asset sellers and buyers in the AM, and the interactions between consumers and producers in the DM. However, the two stages cannot be solved separately. This is because, firstly, in the AM, asset sellers' strategies and (consequently) asset buyers' beliefs depend on what they think producers' beliefs will be like in the following DM, since producers' beliefs affect asset's continuation value and by extension asset sellers' payoff in the *AM*. Secondly, producers' beliefs in the DM also depend on what they think asset buyers' beliefs were like in the preceding AM, since asset buyers' beliefs affect asset sellers' strategies and asset portfolios that the producers observe at the beginning of the DM.

To summarize, compared to the simplified model described above, the main differences are (a) both asset buyers and producers must form beliefs; and (b) the outcomes in the AM and DM are affected by the beliefs of *both* asset buyers and producers. To adapt the definition in Mailath et al. (1993) to our environment, we simply change requirement 3 so that "unreasonable" beliefs from either asset buyers or producers can be the reason why a strategy profitable for some consumers is not played in equilibrium. Requirements 1 and 2 remain unchanged. It is straightforward to see that the logic behind the refinement remains unchanged with our modification.

Appendix B Proofs

Proof of Proposition 1: Firstly, condition (1) ensures that it is optimal for asset buyers in the AM and producers in the DM to accept C-type consumers' offers conditional on asset buyers' and producers' beliefs, thus ensuring that all asset buyers' and producers' strategies are optimal . Second, condition (2) guarantees that asset buyers' beliefs on the equilibrium path, $\gamma^a(\psi^{jk}, s^{jk})$, are consistent with C-type consumers' strategies in the AM. Specifically, if an offer provides C-types with high-quality assets higher surplus compared $\underline{v}^{hO}$ but provides C-types with low-quality assets lower surplus compared $\underline{v}^{lO}$, then asset buyers believe that this offer comes from a C-type with high-quality assets. If the opposite is true, asset buyers believe that the offer comes from a C-type with high-quality assets. If an offer provides C-types with high-quality assets higher surplus compared $\underline{v}^{hO}$, and it also provides C-types with low-quality assets higher surplus compared $\underline{v}^{lO}$, asset buyers believe that both types are making the same offer. Note that since we focus on only pure strategy equilibria, in the AM, the offers can either be pooling (i.e., $(\psi^{lO}, s^{lO}) = (\psi^{hO}, s^{hO})$) or separating (i.e., $(\psi^{lO}, s^{lO}) \neq (\psi^{hO}, s^{hO})$).

Condition (3) requires $\gamma^g(\hat{q}, \hat{z}, \hat{a}; \tilde{z}, \tilde{a})$ to be consistent with C-type consumers' strategies in the DM. Firstly, similar to condition (2), producers believe that an offer comes from a C-type with high-quality assets if and only if C-types with high-quality assets are the sole type that have the incentive to make such an offer. Secondly, because producers can observe consumers' portfolios both before and after the AM, they can infer the quality of their assets.

(a) Because asset buyers' beliefs must satisfy Bayes' rule, in the AM only C-types with high-quality assets (either in transparent meetings or opaque meetings) may sell at a price that is strictly higher than $\rho\delta_h + (1-\rho)\delta_l$. To see why, note that C-types with low-quality assets in transparent meetings can at most sell at $\psi = \delta_l$. Next, if C-types in opaque meetings make pooling offers, they can at most sell at $\psi = \rho\delta_h + (1-\rho)\delta_l$, which is the average asset quality in opaque meetings. If C-types in opaque meetings make separating offers, those with low-quality assets can at most sell at $\psi = \delta_l$.

(b) The belief in (a) ensures that C-types with high-quality assets in transparent meetings will not sell at price less that $\rho\delta_h + (1-\rho)\delta_l$. This in turn ensures that only C-types in opaque meetings may sell at a price that is strictly higher than δ_l but less than $\rho\delta_h + (1-\rho)\delta_l$, which happens only when they make pooling offers. Hence, if $\tilde{\psi} > \delta_l$, the consumers must have been in opaque meetings in the AM and made pooling offers.

(c) The belief in (b) ensures that C-types in opaque meetings will not sell at price less that δ_l when they choose to make pooling offers. Hence, if $\tilde{\psi} \leq \delta_l$, then the consumer must have low-quality assets.

Finally, it is clear that under condition (4), no consumers have the strict incentive to deviate and make an offer not in the set of equilibrium offers $\{(\psi^{jk}, s^{jk}; \hat{q}^{jk}, \hat{z}^{jk}, \hat{a}^{jk})\}$. In other words, the condition on off-equilibrium is sufficient to ensure that $\{(\psi^{jk}, s^{jk}; \hat{q}^{jk}, \hat{z}^{jk}, \hat{a}^{jk})\}$ is part of a PBE. □

Proof of Proposition 2: We show how the proposed strategies are derived, and why they constitute a unique set of undefeated equilibria. Consider the following problem:

$$\max_{\psi, s, \hat{q}, \hat{z}, \hat{a}} u(\hat{q}) + \psi s - \hat{z} - \delta_h(\hat{a} + s) \tag{B.1}$$

$$\text{s.t. } \hat{q} = \hat{z} + \rho\delta_h\hat{a}; \tag{B.2}$$

$$\hat{z} \leq z_s + \psi s; \tag{B.3}$$

$$\psi = \rho\delta_h; \tag{B.4}$$

$$\psi s \leq z_b; \tag{B.5}$$

$$s \leq a; \tag{B.6}$$

$$\hat{a} \leq a - s. \tag{B.7}$$

Denote the solution as (a) $\psi^{hT} = \delta_h$, (b) $s^{hT} = (0, \min\{z_b/\delta_h, (q^* - z_s)/\delta_h, a\}]$, (c) $\hat{q}^{hT} = \min\{q^*, z_s + \delta_h a\}$, (d) $\hat{z}^{hT} = z_s + \psi^{hT} s^{hT}$, and (e) $\hat{a}^{hT} = \frac{\hat{q}^{hT} - \hat{z}^{hT}}{\delta_h}$. Next, consider the following problem:

$$\max_{\psi, s, \hat{q}, \hat{z}, \hat{a}} u(\hat{q}) + \psi s - \hat{z} - \delta_h(\hat{a} + s) \tag{B.8}$$

$$\text{s.t. } \hat{q} = \hat{z} + \rho\delta_h \hat{a}; \tag{B.9}$$

$$\hat{z} \leq z_s + \psi s; \tag{B.10}$$

$$\psi = \rho\delta_h; \tag{B.11}$$

$$\psi s \leq z_b; \tag{B.12}$$

$$s \leq a; \tag{B.13}$$

$$\hat{a} \leq a - s. \tag{B.14}$$

Denote the solution as (a) $\psi_p = \rho\delta_h$; (b) $\hat{q}_p = \max\{z_s, \min\{z_s + \rho\delta_h a, \tilde{q}\}\}$, where $\tilde{q}$ solves $u'(\tilde{q}) = \frac{1}{\rho}$; (c) $s_p = (0, \min\{z_b/(\rho\delta_h), (\hat{q}_p - z_s)/(\rho\delta_h)\}]$, (d) $\hat{z}_p = z_s + \psi_p s_p$, and (e) $\hat{a}_p = \frac{\hat{q}_p - \hat{z}_p}{\rho\delta_h}$.

Now, we show $(\psi_p, s_p, \hat{q}_p, \hat{z}_p, \hat{a}_p)$ and $(\psi^{hT}, s^{hT}, \hat{q}^{hT}, \hat{z}^{hT}, \hat{a}^{hT})$ constitute a unique set of undefeated equilibria. First, type-lO and type-hO consumers must pool in the AM and DM, because otherwise producers will recognize that type-lO consumers' assets are worthless. Second, type-lO and type-hO consumers cannot mimic type-hT consumers because $\psi^{hT} \neq \psi_p$ so $(z_s + \psi_p s_p, a - s_p) \neq (z_s + \psi^{hT} s^{hT}, a - s^{hT})$ for any choices of s_p and s^{hT}. Then, there does not exist an equilibrium where type-hT and/or type-hO consumers are strictly better off since problems (B.1) and (B.8) maximize type-hT and type-hO consumers' surpluses, respectively. This also means that any other equilibria are defeated by the proposed equilibrium, because consumers with high-quality assets are better off in the proposed equilibrium. Finally, type-lT can only use money in the DM since they are the only agents who enter the DM with a portfolio of (z_s, a). □

Proof of Proposition 3: First, define

$$G_1(z) = \lambda\{(1-\rho)\tau[u'(z) - 1] + \rho\tau[u'(\min\{z + \delta_h a, q^*\}) - 1] + (1-\tau)[u'(z + \rho\delta_h a) - 1]\}; \tag{B.15}$$

$$G_2(z) = \lambda\{(1-\rho)\tau[u'(z) - 1] + \rho\tau[u'(\min\{z + \delta_h a, q^*\}) - 1]\}; \tag{B.16}$$

$$G_3(z) = \lambda\{(1-\rho)\tau[u'(z) - 1] + \rho\tau[u'(\min\{z + \delta_h a, q^*\}) - 1] + (1-\tau)[u'(z) - 1]\}. \tag{B.17}$$

Based on earlier analysis, we know that $G_1(z)$ is the marginal value of real balances for all $z \leq \tilde{q} - \rho\delta_h a$; $G_2(z)$ is the marginal value of real balances for all $\tilde{q} - \rho\delta_h a < z \leq \tilde{q}$; and $G_3(z)$ is the marginal value of real balances for all $z > \tilde{q}$. Now, define $i_1 = G_1(\tilde{q} - \rho\delta_h a)$. It is clear that for all $z > \tilde{q} - \rho\delta_h a$, we have $G_2(z) < i_1$ and $G_3(z) < i_1$. In other words, for all $i \geq i_1$, z solves (4.3).

Next, consider $i < i_1$. Define v_1 to be the surplus from holding $z = \tilde{q} - \rho\delta_h a$ units of real balances.

$$v_1(i) = \int_0^{\tilde{q} - \rho\delta_h a} [G_1(z) - i] \, \mathrm{d}z. \tag{B.18}$$

Define $v_2(i)$ to be the surplus from holding $\tilde{q} - \rho\delta_h a < z_2(i) \leq \tilde{q}$ units of real balances where $z_2(i)$ solves $G_2(z_2) = i$ for some $i \in [G_2(\tilde{q}), G_2(\tilde{q} - \rho\delta_h a)]$.

$$v_2(i) = \int_0^{z_2(i)} \{[G_1(z) - i]\mathbf{1}(z \leq \tilde{q} - \rho\delta_h a) + [G_2(z) - i]\mathbf{1}(z > \tilde{q} - \rho\delta_h a)\} \, \mathrm{d}z. \tag{B.19}$$

Similarly, define $v_3(i)$ to be the surplus from holding $z_3(i) > \tilde{q}$ units of real balances where $z_3(i)$ solves

$G_3(z_3) = i$ for some $i \in (0, G_3(\tilde{q})]$.

$$v_3(i) = \int_0^{z_3(i)} \{[G_1(z) - i]\mathbf{1}(z \le \tilde{q} - \rho\delta_h a) + [G_2(z) - i]\mathbf{1}(\tilde{q} - \rho\delta_h a < z \le \tilde{q}) + [G_3(z) - i]\mathbf{1}(z > \tilde{q})\} \, \mathrm{d}z. \tag{B.20}$$

We have two cases to discuss.

Case (1): suppose $G_3(\tilde{q}) \ge G_2(\tilde{q} - \rho\delta_h a)$. Then, $v_3(G_3(\tilde{q})) < v_1(G_3(\tilde{q}))$. Note that

$$v_3(i) - v_1(i) = \int_{\tilde{q}-\rho\delta_h a}^{z_3(i)} \{[G_2(z) - i]\mathbf{1}(z \le \tilde{q}) + [G_3(z) - i]\mathbf{1}(z > \tilde{q})\} \, \mathrm{d}z, \tag{B.21}$$

which is decreasing in i. Hence, there must exist $\tilde{i} < i_1$ such that $v_3(\tilde{i}) = v_1(\tilde{i})$.

Suppose $\tilde{i} \ge G_2(\tilde{q} - \rho\delta_h a)$. Then $v_2(i) < v_3(i)$ for all $i \in [G_2(\tilde{q}), G_2(\tilde{q} - \rho\delta_h a))$, because $v_2(G_2(\tilde{q} - \rho\delta_h a)) = v_1(G_2(\tilde{q} - \rho\delta_h a))$ and

$$v_3(i) - v_2(i) = \int_{z_2(i)}^{z_3(i)} \{[G_2(z) - i]\mathbf{1}(z_2(i) \le z \le \tilde{q}) + [G_3(z) - i]\mathbf{1}(z > \tilde{q})\} \, \mathrm{d}z \tag{B.22}$$

is also decreasing in i since $G_2(z) - i < 0$ for all $z > z_2(i)$. In this case, let $i_2 = i_3 = \tilde{i}$. Then, for all $i_2 \le i < i_1$, $z = \tilde{q} - \rho\delta_h a$. For all $i \le i_3$, z solves (4.5).

Suppose $\tilde{i} < G_2(\tilde{q} - \rho\delta_h a)$. Then there must exist $i_3 < G_2(\tilde{q} - \rho\delta_h a)$ such that $v_3(i_3) = v_2(i_3)$. Let $i_2 = G_2(\tilde{q} - \rho\delta_h a)$, then for all $i_2 \le i < i_1$, $z = \tilde{q} - \rho\delta_h a$. For all $i_3 < i < i_2$, z solves (4.4). Lastly, for all $i \le i_3$, z solves (4.5).

Case (2): suppose $G_3(\tilde{q}) < G_2(\tilde{q} - \rho\delta_h a)$. Then, for all $i \in [G_3(\tilde{q}), G_2(\tilde{q} - \rho\delta_h a)]$, we have $v_2(i) > v_1(i)$. Now, consider $i < G_3(\tilde{q})$. Note that $v_3(G_3(\tilde{q})) < v_2(G_3(\tilde{q}))$, and that

$$v_3(i) - v_2(i) = \int_{z_2(i)}^{z_3(i)} \{[G_2(z) - i]\mathbf{1}(z_2(i) \le z \le \tilde{q}) + [G_3(z) - i]\mathbf{1}(z > \tilde{q})\} \, \mathrm{d}z, \tag{B.23}$$

which is decreasing in i. Hence, there must exist $i_3 < G_2(\tilde{q} - \rho\delta_h a)$ such that $v_3(i_3) = v_2(i_3)$. Now, let $i_2 = G_2(\tilde{q} - \rho\delta_h a)$, then for all $i_2 \le i < i_1$, $z = \tilde{q} - \rho\delta_h a$. For all $i_3 < i < i_2$, z solves (4.4). Lastly, for all $i \le i_3$, z solves (4.5).

Now we prove the case where $\rho\delta_h a \ge \tilde{q}$. Since by assumption we have $u'(0) = \infty$, then $G_2(0) = \infty$. Following the proof of Proposition 3, we know that there exists $i^\dagger < \infty$ such that $\tilde{v}_3(i^\dagger) = \tilde{v}_2(i^\dagger)$, where

$$\tilde{v}_2(i) = \int_0^{z_2(i)} [G_2(z) - i] \, \mathrm{d}z, \tag{B.24}$$

and

$$\tilde{v}_3(i) = \int_0^{z_3(i)} \{[G_2(z) - i]\mathbf{1}(z \le \tilde{q}) + [G_3(z) - i]\mathbf{1}(z > \tilde{q})\} \, \mathrm{d}z. \tag{B.25}$$

□

Proof of Proposition 4: I prove the comparative statics of z with respect to i and τ. The rest of the comparative statics follows directly from Proposition 2. First, assume $\rho\delta_h a < \tilde{q}$. If $i \geq i_1$, z solves

$$i = \lambda\{(1-\rho)\tau[u'(z)-1] + \rho\tau[u'(\min\{z+\delta_h a, q^*\})-1] + (1-\tau)[u'(z+\rho\delta_h a)-1]\}. \tag{B.26}$$

It is straightforward to see that z is decreasing in i. Now, take derivative with respect to τ to get

$$\frac{\partial z}{\partial \tau} = -\frac{(1-\rho)[u'(z)-1] + \rho[u'(\min\{z+\delta_h a, q^*\})-1] - [u'(z+\rho\delta_h a)-1]}{(1-\rho)\tau u''(z) + \rho\tau u''(z+\delta_h a)\mathbf{1}(z+\delta_h a < q^*) + (1-\tau)u''(z+\rho\delta_h a)} > 0, \tag{B.27}$$

because $(1-\rho)[u'(z)-1] + \rho[u'(\min\{z+\delta_h a, q^*\})-1] \geq (1-\rho)[u'(z)-1] + \rho[u'(z+\delta_h a)-1] \geq u'(z+\rho\delta_h a) - 1$ as long as $u'''(.) > 0$. Next, if $i_2 \leq i < i_1$, $z = \tilde{q} - \rho\delta_h a$. Hence, z is unaffected by changes in i and τ. If $i_3 < i < i_2$, z solves

$$i = \lambda\{(1-\rho)\tau[u'(z)-1] + \rho\tau[u'(\min\{z+\delta_h a, q^*\})-1]\}. \tag{B.28}$$

Again, z is decreasing in i. The derivative with respect to τ is

$$\frac{\partial z}{\partial \tau} = -\frac{(1-\rho)[u'(z)-1] + \rho[u'(\min\{z+\delta_h a, q^*\})-1]}{(1-\rho)\tau u''(z) + \rho\tau u''(z+\delta_h a)\mathbf{1}(z+\delta_h a < q^*)} > 0. \tag{B.29}$$

Finally, if $i \leq i_3$, z solves

$$i = \lambda\{(1-\rho)\tau[u'(z)-1] + \rho\tau[u'(\min\{z+\delta_h a, q^*\})-1] + (1-\tau)[u'(z)-1]\}, \tag{B.30}$$

so z is decreasing in i. The derivative with respect to τ is

$$\frac{\partial z}{\partial \tau} = -\frac{(1-\rho)[u'(z)-1] + \rho[u'(\min\{z+\delta_h a, q^*\})-1] - [u'(z)-1]}{(1-\rho)\tau u''(z) + \rho\tau u''(z+\delta_h a)\mathbf{1}(z+\delta_h a < q^*) + (1-\tau)u''(z)} < 0. \tag{B.31}$$

Now, assume $\rho\delta_h a \geq \tilde{q}$. Recall that for all $i > i^\dagger$, z solves (4.4), and for all $i \leq i^\dagger$, z solves (4.5). Hence, the results follow from the above arguments.□

Proof of Proposition 5: We start with the DM. For the moment, let us assume that $\tau = 0$. In this case, there are two types of equilibrium in the DM.
Case 1: separating equilibrium. Suppose that the DM equilibrium is separating. Let the offer be $(\hat{q}, \hat{z}, \hat{a})$. Consumers with high-quality assets solve

$$\max_{\hat{q},\hat{z},\hat{a}} u(\hat{q}) - \hat{z} - \delta_h \hat{a} \tag{B.32}$$

$$\text{s.t. } \hat{z} + \delta_h \hat{a} \geq \hat{q}, \tag{B.33}$$

$$v^* \geq u(\hat{q}) - \hat{z} - \delta_l \hat{a}, \tag{B.34}$$

$$\hat{z} \leq \tilde{z},\ \hat{a} \leq \tilde{a}, \tag{B.35}$$

where v^* is given by

$$v^* = \max_{\hat{q},\hat{z},\hat{a}} u(\hat{q}) - \hat{z} - \delta_l \hat{a} \tag{B.36}$$

$$\text{s.t. } \hat{z} + \delta_l \hat{a} = \hat{q}, \tag{B.37}$$

$$\hat{z} \leq \tilde{z},\ \hat{a} \leq \tilde{a}. \tag{B.38}$$

Let the solutions to the first problem and the second problem be (q_h, z_h, a_h) and (q_l, z_l, a_l), respectively. We have $q_l = \min\{\tilde{z}+\delta_l\tilde{a}, q^*\}$. Next, we show that (B.33) and (B.34) must bind at (q_h, z_h, a_h). Suppose (B.33) is strict, let $(\hat{q}', \hat{z}', \hat{a}')$ be such that $\hat{z}' = z_h$, $\hat{a}' < a_h$, and

$$u(q_h) - z_h - \delta_l a_h = u(\hat{q}') - \hat{z}' - \delta_l \hat{a}'. \tag{B.39}$$

Then, it is easy to show that $u(q_h) - z_h - \delta_h a_h < u(\hat{q}') - \hat{z}' - \delta_h \hat{a}'$, a contradiction. Next, suppose (B.34) is strict, then because $q_h < q_l$, the consumer can increase a_h and obtain higher utility without giving consumers with low-quality assets the incentive to deviate. Hence, it must be that (B.34) also binds. We can then rewrite the first problem as

$$\max_{\hat{z},\hat{a}} u(\hat{z} + \delta_h \hat{a}) - \hat{z} - \delta_h \hat{a} \tag{B.40}$$

$$\text{s.t. } u(q_l) - q_l = u(\hat{z} + \delta_h \hat{a}) - \hat{z} - \delta_l \hat{a}, \tag{B.41}$$

$$\hat{z} \leq \tilde{z}. \tag{B.42}$$

Since $\tilde{z} < q^*$ by assumption, we can conclude that $z_h = \tilde{z}$. And a_h solves

$$u(q_l) - q_l = u(\tilde{z} + \delta_h a_h) - \tilde{z} - \delta_l a_h. \tag{B.43}$$

It is straightforward to show that a_h is decreasing in $\tilde{z}$, and that $\tilde{z} + \delta_h a_h$ is increasing in $\tilde{z}$.
Case 2: pooling equilibrium. Suppose that the DM equilibrium is pooling. Consumers with high-quality assets solve

$$\max_{\hat{q},\hat{z},\hat{a}} u(\hat{q}) - \hat{z} - \delta_h \hat{a} \tag{B.44}$$

$$\text{s.t. } \hat{z} + \bar{\delta}\hat{a} = \hat{q}, \tag{B.45}$$

$$u(\hat{q}) - \hat{z} - \delta_l \hat{a} \geq u(q^*) - q^*, \tag{B.46}$$

$$\hat{z} \leq \tilde{z}, \quad \hat{a} \leq \tilde{a}, \tag{B.47}$$

where $\bar{\delta} = \rho\delta_h + (1-\rho)\delta_l$ is the average asset quality. Note that (B.46) is the participation constraint for consumers with low-quality assets. Let the solution be (q_p, z_p, a_p). Define $\tilde{q}$ to be such that $u'(\tilde{q}) = \frac{\delta_h}{\bar{\delta}}$. If $\tilde{z} \geq \tilde{q}$, pooling cannot happen because consumers with high-quality assets will prefer not selling any assets. Hence, (B.46) is not satisfied. Now, suppose $\tilde{z} < \tilde{q}$. If (B.46) does not bind, we have $q_p = \tilde{q}$, $z_p = \tilde{z}$, and $a_p = (q_p - z_p)/\bar{\delta}$. Since a_p is decreasing in $\tilde{z}$, (B.46) binds when $\tilde{z}$ is sufficiently large. In such case, it is clear that consumers with high-quality assets will prefer separating equilibrium.

Finally, we solve for the equilibrium in the AM. Consider a consumer with high-quality assets. She may sell some assets at the pooling price in the AM and make a separating offer in the DM, or she may make a pooling offer in the AM. In the latter case, she does not have the incentive to sell in the AM. This is because that selling at the pooling price does not increase her surplus. However, unless consumers sell all of their assets in the AM, selling in the AM makes (B.46) more likely to bind because $\tilde{z}$ is larger.

Now, consider the first case and let $s \geq 0$ denote the assets sold in the AM. Consider a consumer with high-quality assets. In the DM, the consumer will have $z_s + \bar{\delta}s$ units of real balances. We can conclude that the consumer will offer $\hat{z} = z_s + \bar{\delta}s$ in the DM, because otherwise the consumer can increase her utility by lowering s. For now, let us assume that trade in the AM is not constrained by asset buyers' money holdings z_b. The consumer solves

$$\max_{s,\hat{a}} u(z_s + \bar{\delta}s + \delta_h \hat{a}) - z_s - \delta_h s - \delta_h \hat{a} \tag{B.48}$$

$$\text{s.t. } u(\min\{z_s + \bar{\delta}s + \delta_l(a-s), q^*\}) - \min\{z_s + \bar{\delta}s + \delta_l(a-s), q^*\}$$
$$= u(z_s + \bar{\delta}s + \delta_h\hat{a}) - z_s - \bar{\delta}s - \delta_l\hat{a}, \tag{B.49}$$
$$s + a \leq a. \tag{B.50}$$

Note that as long as $s < a$, then $\hat{a} > 0$. To see why, note that it must be that $z_s + \bar{\delta}s < q^*$, otherwise consumers' with high-quality assets can reduce s and increase their utility. If $s < a$, consumers with high-quality assets will have assets available to use directly as payment in the DM. Because $z_s + \bar{\delta}s < q^*$, a separating offer in the DM that includes both money and assets (i.e., $\hat{a} > 0$) increases the utility of consumers with high-quality assets. If $s = 0$, consumers makes separating offers in the DM but do not sell in the AM. To conclude, consumers make separating offers in the DM unless $s = a$.

Using (B.49), we can define $\hat{a}$ as a function of s, $\hat{a}(s)$. We have

$$\hat{a}'(s) = \frac{(\bar{\delta} - \delta_l)[u'(q_l) - 1] - \bar{\delta}[u'(q_h) - 1]}{\delta_h u'(q_h) - \delta_l}, \tag{B.51}$$

where $q_l = \min\{z_s + \bar{\delta}s + \delta_l(a-s), q^*\}$ and $q_h = z_s + \bar{\delta}s + \delta_h\hat{a}(s)$. Then, the first derivative of (B.48) with respect to s is given by

$$\bar{\delta}u'(q_h) - \delta_h + \delta_h[u'(q_h) - 1]\hat{a}'(s) \propto \frac{[(1-\rho)\delta_l + (2\rho - 1)\delta_h]u'(q_h)}{\rho\delta_h} + [u'(q_l) - 1][u'(q_h) - 1] - 1. \tag{B.52}$$

In what follows, we assume $(1-\rho)\delta_l + (2\rho-1)\delta_h > 0$. Note that q_l is increasing in s as long as $q_l < q^*$. In addition,

$$\frac{\mathrm{d}(\bar{\delta}s + \delta_h a)}{\mathrm{d}s} = \frac{\bar{\delta}\left(1 - \frac{\delta_l}{\delta_h}\right) + (\bar{\delta} - \delta_l)[u'(q_l) - 1]}{u'(q_h) - \frac{\delta_l}{\delta_h}} > 0, \tag{B.53}$$

so q_h is increasing in s as well. Hence, we can conclude that (B.52) is decreasing in s. Then, there is a unique solution to (B.48). Furthermore, $s = 0$ if

$$\frac{[(1-\rho)\delta_l + (2\rho-1)\delta_h]u'(z_s + \delta_h\hat{a}(0))}{\rho\delta_h} + [u'(\min\{z_s + \delta_l a, q^*\}) - 1][u'(z_s + \delta_h\hat{a}(0)) - 1] - 1 \leq 0, \tag{B.54}$$

and $s = a$ if

$$\frac{[(1-\rho)\delta_l + (2\rho-1)\delta_h]u'(z_s + \bar{\delta}a)}{\rho\delta_h} + [u'(z_s + \bar{\delta}a) - 1]^2 - 1 \geq 0. \tag{B.55}$$

Now define z_s'' to be such that (B.54) holds at equality and z_s' to be such that (B.55) holds at equality. We can then conclude that $s = a$ if $z_s < z_s'$ and $s = 0$ if $z_s \geq z_s''$. Note that for (B.55) to hold, it must be that $u'(z_s + \bar{\delta}a) > \frac{\delta_h}{\bar{\delta}}$, otherwise $z_s' = 0$, and consumers do not make pooling offers in the DM. If $z_s' \leq z < z_s''$, s solves

$$\frac{[(1-\rho)\delta_l + (2\rho-1)\delta_h]u'(q_h)}{\rho\delta_h} + [u'(q_l) - 1][u'(q_h) - 1] - 1 = 0. \tag{B.56}$$

In this case, an increase in z_s will have no effect on q_l and q_h, because otherwise (B.49) and (B.56) will not hold at the same time. This implies that $z_s + \bar{\delta}s$ will remain unchanged. Finally, suppose $z_s' \leq z_s < z_s''$. Consumers may be constrained in the AM by asset buyers' money holdings z_b. In other words,

$$\frac{[(1-\rho)\delta_l + (2\rho - 1)\delta_h]u'(q_h)}{\rho\delta_h} + [u'(q_l) - 1][u'(q_h) - 1] - 1 > 0 \tag{B.57}$$

if $\bar{\delta}s = z_b$.

Finally, if $\tau > 0$, so some consumers are in transparent meetings. If such a consumer has high-quality assets, she will be able to sell at a price equal to δ_h. If such a consumer has low-quality assets, she has to sell at a price equal to δ_l. Consumers in opaque meetings will also sell a positive amount of assets at price $\bar{\delta}$. This means that in the DM, the latter cannot mimic the former or consumers in opaque meetings. □

Proof of Proposition 6: Consider the marginal value of real balances at the beginning of the AM for consumers in opaque meetings. First, consider consumers in transparent meetings. If they have high-quality assets, then the marginal value is given by $u'(\max\{z_s + \delta_h a_s, q^*\})$. If they have low-quality assets, then the marginal value is given by $u'(\max\{z_s + \delta_l a_s, q^*\})$. Next, consider consumers in opaque meetings. We have several cases to discuss.
(1) Suppose that $z_s \geq z_s''$. The marginal value of real balances is one for consumers with low-quality assets. The marginal value of real balances for consumers with high-quality assets is given by

$$\frac{\{(\delta_h - \delta_l) + \delta_h[u'(\min\{z_s + \delta_l a, q^*\}) - 1]\}[u'(z_s + \delta_h a) - 1]}{\delta_h u'(z_s + \delta_h a) - \delta_l} + 1. \tag{B.58}$$

(2) Suppose that $z_s' \leq z_s < z_s''$. The marginal value of real balances is one for consumers with low-quality assets. For consumers with high-quality assets, the marginal value of real balances may depend on z_b as well. For simplicity, we assume that $z_s' \geq \bar{\delta}a$. This will guarantee that $z_s'' < 2z_s'$, which means that in a symmetric equilibrium, consumers are not constrained by z_b. Note that $z_s' > \bar{\delta}a$ implies $z_s'' < 2z_s'$ because $z_s' + \bar{\delta}a > z_s'' + \delta_h a(0)$. Finally, if consumers are not constrained by z_b, the marginal value of real balances will be one.
(3) Suppose that $z_s < z_s'$. The marginal value of real balances for both types of consumers is given by $u'(z_s + \bar{\delta}a)$.

In what follows, we focus only on symmetric equilibrium where all agents choose the same money holding in the CM. That is, $z_s = z_b = z$. We show how the marginal value of real balances in this scenario depends on z in the following figure.

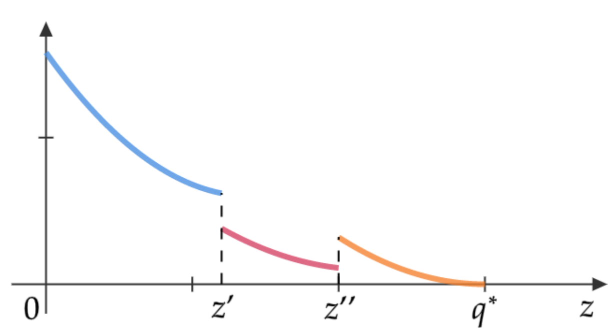


Figure 15: Marginal value of z

Then, by following the proof of Proposition 3, it is straightforward to show that there exist μ' and μ'' such that if $\mu > \mu''$, the optimal z will be smaller than z'. If $\mu' < \mu \leq \mu''$, the optimal z will be between z' and z''. Finally, if $\mu \leq \mu'$, the optimal z will be larger than z''. □

Proof of Lemma 2: First, assume $a < \frac{\tilde{q}}{\rho\delta_h}$. Suppose $z < \tilde{q} - \rho\delta_h a$. We have

$$\mathcal{B}_1'(z) = (1-\rho)u'(z) + \rho u'(\min\{z + \delta_h a, q^*\}) - u'(z + \rho\delta_h a). \tag{B.59}$$

If $z + \delta_h a \leq q^*$, then it is clear that $\mathcal{B}_1'(z) > 0$ since $u'''(.) > 0$. Now, suppose $z + \delta_h a > q^*$. Note that $q^* - \delta_h a - (\tilde{q} - \rho\delta_h a) = q^* - \tilde{q} - (1-\rho)\delta_h a$ is decreasing in a. Suppose $\tilde{q} = \rho\delta_h a$, then $q^* - \delta_h a - (\tilde{q} - \rho\delta_h a) = q^* - \delta_h a > 0$ because $\delta_h a < q^*$ by assumption. In other words, $q^* - \delta_h a - (\tilde{q} - \rho\delta_h a) > 0$ when $a < \min\left\{\frac{\tilde{q}}{\rho\delta_h}, \frac{q^*}{\delta_h}\right\}$. This implies that if $z + \delta_h a > q^*$, then $z + \rho\delta_h a > \tilde{q}$. In other words, for all $q^* - \delta_h a < z \leq \tilde{q}$, we have

$$\mathcal{B}_1'(z) = (1-\rho)[u'(z) - 1] + \rho[u'(\min\{z + \delta_h a, q^*\}) - 1] > 0. \tag{B.60}$$

Note also that when $z = \tilde{q}$, $\mathcal{B}_1(z) > 0$. However, if $z \leq \tilde{q} - \rho\delta_h a$, $\mathcal{B}_1(z) < 0$ because $u''(.) < 0$. Hence, there exists $\tilde{q} - \rho\delta_h a < \tilde{z} < \tilde{q}$ such that $\mathcal{B}_1(\tilde{z}) = 0$. Finally, suppose $\tilde{q} < z < q^*$. We have

$$\mathcal{B}_1'(z) = \rho[u'(\min\{z + \delta_h a, q^*\}) - u'(z)] < 0. \tag{B.61}$$

Second, suppose $a \geq \frac{\tilde{q}}{\rho\delta_h}$. We have

$$\mathcal{B}_1'(z) = (1-\rho)[u'(z) - 1] + \rho[u'(\min\{z + \delta_h a, q^*\}) - 1] > 0 \tag{B.62}$$

for all $z \leq \tilde{q}$, and

$$\mathcal{B}_1'(z) = \rho[u'(\min\{z + \delta_h a, q^*\}) - u'(z)] < 0 \tag{B.63}$$

for all $\tilde{q} < z < q^*$. Note that $\mathcal{B}_1(z) < 0$ when $z = 0$, but $\mathcal{B}_1(z) > 0$ when $z = \tilde{q}$. Hence, there exists $0 < \tilde{z} < \tilde{q}$ such that $\mathcal{B}_1(\tilde{z}) = 0$. □

Proof of Proposition 7: Assume $a < \frac{\tilde{q}}{\rho\delta_h}$. From Proposition 2, when $z < \tilde{q} - \rho\delta_h a$, the marginal value of money is $u'(z + \rho\delta_h a)$. Since $\kappa < \mathcal{B}_1(\tilde{z})$, there exists $\tilde{z} < z^\dagger < \tilde{q}$ such that $\mathcal{B}_1(z^\dagger) = \kappa$. Then, when $\tilde{q} - \rho\delta_h a \leq z \leq z^\dagger$, the marginal value of money is 1. Since $\mathcal{B}_1'(z) < 0$ for all $\tilde{q} < z < q^*$ and $\mathcal{B}_1(q^*) = 0$, there exists $z^\ddagger > \tilde{q}$ such that $\mathcal{B}_1(z^\ddagger) = \kappa$. Hence, when $z^\dagger < z < z^\ddagger$, agents pay κ, and the marginal value of money is $(1-\rho)u'(z) + \rho u'(\min\{z + \delta_h a, q^*\})$. Finally, when $z^\ddagger \leq z \leq q^*$, the marginal value of money is $u'(z)$.

Now, define $G_1(z) = \lambda[u'(z + \rho\delta_h a) - 1]$, $G_2(z) = \lambda\{(1-\rho)[u'(z) - 1] + \rho[u'(\min\{z + \delta_h a, q^*\}) - 1]\}$, and $G_3(z) = \lambda[u'(z) - 1]$. Next, let $i_1 = G_1(\tilde{q} - \rho\delta_h a)$. It is clear that for all $z > \tilde{q} - \rho\delta_h a$, we have $G_2(z) < i_1$ and $G_3(z) < i_1$. In other words, for all $i < i_1$, z solves $G_1(z) = i$ where $i = \frac{1+\mu}{\beta} - 1$.

Next, consider $i \leq i_1$. Define v_1 to be the surplus from holding $z = \tilde{q} - \rho\delta_h a$ units of real balances.

$$v_1(i) = \int_0^{\tilde{q} - \rho\delta_h a} [G_1(z) - i]\, \mathrm{d}z. \tag{B.64}$$

Define $v_2(i)$ to be the surplus from holding $\tilde{q} - \rho\delta_h a < z_2(i) < z^\ddagger$ units of real balances, where $z_2(i)$

solves $G_2(z) = i$ for some $i \in [0, G_2(z^\dagger)]$.

$$v_2(i) = \int_0^{z_2(i)} \{[G_1(z) - i]\mathbf{1}(z < \tilde{q} - \rho\delta_h a) - i\mathbf{1}(\tilde{q} - \rho\delta_h a \leq z \leq z^\dagger) + [G_2(z) - i]\mathbf{1}(z^\dagger < z < z^\ddagger)\}\,\mathrm{d}z. \tag{B.65}$$

Similarly, define $v_3(i)$ to be the surplus from holding $z_3(i) > z^\ddagger$ units of real balances where $z_3(i)$ solves $G_3(z_3) = i$ for some $i \in [0, G_3(\tilde{q})]$.

$$v_3(i) = \int_0^{z_3(i)} \{[G_1(z) - i]\mathbf{1}(z < \tilde{q} - \rho\delta_h a) - i\mathbf{1}(\tilde{q} - \rho\delta_h a \leq z \leq z^\dagger) + [G_2(z) - i]\mathbf{1}(z^\dagger < z < z^\ddagger) + [G_3(z) - i]\mathbf{1}(z \geq z^\ddagger)\}\,\mathrm{d}z. \tag{B.66}$$

Now, consider $v_2(i) - v_1(i)$ for all $i \in [0, G_2(z^\dagger)]$.

$$v_2(i) - v_1(i) = \int_{\tilde{q}-\rho\delta_h a}^{z_2(i)} \{-i\mathbf{1}(\tilde{q} - \rho\delta_h a \leq z \leq z^\dagger) + [G_2(z) - i]\mathbf{1}(z^\dagger < z < z^\ddagger)\}\,\mathrm{d}z. \tag{B.67}$$

It is clear that $v_2(i) - v_1(i)$ is decreasing in i and there exists some $\tilde{i}_2 \in (0, G_2(z^\dagger))$ such that $v_2(\tilde{i}_2) - v_1(\tilde{i}_2) = 0$. Next, consider $v_3(i) - v_2(i)$ for all $i \in [G_2(z^\ddagger), \min\{G_2(z^\dagger), G_3(z^\ddagger)\}]$.

$$v_3(i) - v_2(i) = \int_{z_2(i)}^{z_3(i)} \{[G_2(z) - i]\mathbf{1}(z_2(i) < z < z^\ddagger) + [G_3(z) - i]\mathbf{1}(z \geq z^\ddagger)\}\,\mathrm{d}z, \tag{B.68}$$

which is decreasing in i, and there exists some $\tilde{i}_3 \in (G_2(z^\ddagger), \min\{G_2(z^\dagger), G_3(z^\ddagger)\})$ such that $v_3(\tilde{i}_3) - v_2(\tilde{i}_3) = 0$ provided that $v_3(\min\{G_2(z^\dagger), G_3(z^\ddagger)\}) - v_2(\min\{G_2(z^\dagger), G_3(z^\ddagger)\}) < 0$. Now, we have two cases to discuss.
Case I: Suppose $\tilde{i}_2 \geq \tilde{i}_3 > G_2(z^\ddagger)$. Then, let $i_2 = \tilde{i}_2$ and $i_3 = \tilde{i}_3$. For all $i_2 \leq i \leq i_1$, $z = \tilde{q} - \rho\delta_h a$. For all $i_3 < i < i_2$, z solves $G_2(z) = i$. For all $i \leq i_3$, z solves $G_3(z) = i$.
Case II: Suppose $\tilde{i}_3 > \tilde{i}_2$ or $v_3(\min\{G_2(z^\dagger), G_3(z^\ddagger)\}) - v_2(\min\{G_2(z^\dagger), G_3(z^\ddagger)\}) \geq 0$. Consider $v_3(i) - v_1(i)$ for all $i \leq G_3(z^\ddagger)$.

$$v_3(i) - v_1(i) = \int_{\tilde{q}-\rho\delta_h a}^{z_3(i)} \{-i\mathbf{1}(\tilde{q} - \rho\delta_h a \leq z \leq z^\dagger) + [G_2(z) - i]\mathbf{1}(z^\dagger < z < z^\ddagger) + [G_3(z) - i]\mathbf{1}(z \geq z^\ddagger)\}\,\mathrm{d}z, \tag{B.69}$$

which is decreasing in i, and there exists some $\tilde{i}_4 \in (0, G_3(z^\ddagger))$ such that $v_3(\tilde{i}_4) - v_1(\tilde{i}_4) = 0$. In this case, let $i_2 = i_3 = \tilde{i}_4$. Then, for all $i_2 \leq i \leq i_1$, $z = \tilde{q} - \rho\delta_h a$. For all, $i \leq i_3$, z solves $G_3(z) = i$. □

Proof of Proposition 8: I prove the comparative statics of z with respect to i. The rest of the comparative statics follows directly from Proposition 2. If $i \geq i_1$, z solves $i = \lambda[u'(z + \rho\delta_h a) - 1]$. Then it is clear that z is decreasing in i. If $i_2 \leq i < i_1$, $z = \tilde{q} - \rho\delta_h a$, then z is unaffected by i. If $i_3 < i < i_2$, z solves $i = \lambda\{(1-\rho)[u'(z) - 1] + \rho[u'(\min\{z + \delta_h a, q^*\}) - 1]\}$, then it is clear that z is decreasing in i. Finally, if $i \leq i_3$, z solves $i = \lambda[u'(z) - 1]$, so z is decreasing in i. □

Proof of Lemma 3: Suppose $z < q^\dagger(\tilde{\tau}) - \xi(\tilde{\tau})\delta_h a$, we have

$$\frac{\partial \mathcal{B}_2(z, \tilde{\tau})}{\partial z} = u'(z + \delta_h a) - 1 - [u'(z + \xi(\tilde{\tau})\delta_h a) - 1] < 0. \tag{B.70}$$

Suppose $z > q^\dagger(\tilde{\tau})$, we have

$$\frac{\partial \mathcal{B}_2(z,\tilde{\tau})}{\partial z} = u'(\hat{q}^{hT}) - 1 - [u'(z) - 1] < 0. \tag{B.71}$$

Suppose $z \in [q^\dagger(\tilde{\tau}) - \xi(\tilde{\tau})\delta_h a, q^\dagger(\tilde{\tau})]$, we have

$$\frac{\partial \mathcal{B}_2(z,\tilde{\tau})}{\partial z} = u'(z + \delta_h a) - 1 > 0. \tag{B.72}$$

Finally, $\mathcal{B}_2(z,\tilde{\tau})$ is increasing in $\tilde{\tau}$ because $u(\hat{q}(\tilde{\tau})) + z + \rho\delta_h a - \hat{q}(\tilde{\tau})$ is decreasing in $\tilde{\tau}$. $\square$

Proof of Lemma 4: Suppose $\tilde{\tau} \in (0,1)$. Let z^* denote the optimal choice of z. We have

$$u(\hat{q}^{hT}) - \hat{q}^{hT} - [u(\hat{q}(\tilde{\tau}'(z^*))) - \hat{q}(\tilde{\tau}'(z^*))] = \kappa, \tag{B.73}$$

where $\hat{q}^{hT} = \min\{z^*+\delta_h a, q^*\}$, and $\hat{q}(\tilde{\tau}) = \max\{z^*, \min\{z^*+\xi(\tilde{\tau})\delta_h a, q^\dagger(\tilde{\tau})\}\}$ where $u'(q^\dagger(\tilde{\tau})) = 1/\xi(\tilde{\tau})$ and $\xi(\tilde{\tau}) = \frac{\rho(1-\tilde{\tau})}{\rho(1-\tilde{\tau})+1-\rho}$. In other words, conditional on z^* and $\tilde{\tau}$, consumers with high-quality assets are indifferent between paying κ or not. However, ex ante, the surplus when choosing to pay κ in the AM is given by

$$V_1 \equiv \lambda\{(1-\rho)[u(\hat{q}^{lT}) + z^* - \hat{q}^{lT}] + \rho[u(\hat{q}^{hT}) + z^* + \delta_h a - \hat{q}^{hT} - \kappa]\} + (1-\lambda)(z^* + \rho\delta_h a) - (i+1)z^*, \tag{B.74}$$

where $\hat{q}^{lT} = z^*$, while the ex ante surplus when choosing not to pay κ later is given by

$$V_2 \equiv \lambda[u(\hat{q}(\tilde{\tau}'(z^*))) + z^* + \rho\delta_h a - \hat{q}(\tilde{\tau}'(z^*))] + (1-\lambda)(z^* + \rho\delta_h a) - (i+1)z^*. \tag{B.75}$$

It is straightforward to show that $V_1 = V_2$ if and only if $\hat{q}(\tilde{\tau}'(z^*)) = z^*$. Take the derivatives of V_1 and V_2 with respect to z^* and get

$$\lambda\{(1-\rho)[u'(z^*) - 1] + \rho[u'(\min\{q^*, z^* + \delta_h a\}) - 1]\} - i, \tag{B.76}$$

$$\lambda[u'(z^*) - 1] - i. \tag{B.77}$$

Notice that $(1-\rho)[u'(z^*) - 1] + \rho[u(\min\{q^*, z^* + \delta_h a\}) - 1] = \lambda[u'(z^*) - 1]$ if and only if $z^* = q^*$. This implies that a consumer considering not paying κ later in the AM and a consumer considering paying κ would not have chosen the same z. In other words, $\tilde{\tau} \in (0,1)$ is not possible if z is chosen optimally. $\square$

Proof of Proposition 9: Define $G_1(z) = \lambda\{(1-\rho)[u'(z) - 1] + \rho[u'(\min\{q^*, z + \delta_h a\}) - 1]\}$ and

$$G_2(z) = \begin{cases} \lambda[u'(\max\{z, \min\{z + \rho\delta_h a, \tilde{q}\}\}) - 1], & \text{if } z < \tilde{q} - \rho\delta_h a, \\ 0, & \text{if } \tilde{q} - \rho\delta_h a \le z \le \tilde{q}, \\ \lambda[u'(z) - 1], & \text{if } z > \tilde{q}. \end{cases} \tag{B.78}$$

Then, $G_1(z)$ is the marginal value of holding money when a consumer later pays κ in the AM, and $G_2(z)$ is the marginal value of holding money when a consumer later does not κ in the AM There are three scenarios to consider.

(1) Assume that $\kappa \ge \bar{\mathcal{B}}_2(\tilde{q})$. Define z_1 and z_2 to be such that $\underline{\mathcal{B}}_2(z_1) = \kappa$ and $\bar{\mathcal{B}}_2(z_2) = \kappa$. Then, for

all $z \leq z_1$, the marginal value of money is given by $G_1(z)$. For all $z \geq z_2$, the marginal value of money is given by $G_2(z)$. For all $z \in (z_1, z_2)$, the marginal value of money is given by $G_1(z)$ if consumers expect $\tilde{\tau} = 0$, and $G_2(z)$ if consumers expect $\tilde{\tau} = 1$.
(2) Assume that $\underline{\mathcal{B}}_2(\tilde{q} - \rho\delta_h a) < \kappa < \bar{\mathcal{B}}_2(\tilde{q})$. Define z_2 to be such that $\bar{\mathcal{B}}_2(z_2) = \kappa$. Then, there exist z_1 and $z_1 < z_1' < z_2$ such that $\underline{\mathcal{B}}_2(z_1) = \kappa$ and $\underline{\mathcal{B}}_2(z_1') = \kappa$. Then, for all $z \leq z_1$, the marginal value of money is $\lambda\{(1-\rho)[u'(z) - 1] + \rho[u'(\min\{q^*, z + \delta_h a\}) - 1]\}$. For all $z \geq z_1'$, the marginal value of money is $\lambda[u'(\max\{z, \min\{z + \rho\delta_h a, \tilde{q}\}\}) - 1]$. For all $z \in (z_1, z_1')$, the marginal value of money is $\lambda[u'(\max\{z, \min\{z + \rho\delta_h a, \tilde{q}\}\}) - 1]$ if consumers expect $\tilde{\tau} = 0$, and $\lambda\{(1-\rho)[u'(z) - 1] + \rho[u'(\min\{q^*, z + \delta_h a\}) - 1]\}$ if consumers expect $\tilde{\tau} = 1$.
(3) Assume that $\kappa \leq \underline{\mathcal{B}}_2(\tilde{q} - \rho\delta_h a)$. Define z_2 to be such that $\bar{\mathcal{B}}_2(z_2) = \kappa$. Then, for all $z < z_2$, the marginal value of money is $\lambda\{(1-\rho)[u'(z) - 1] + \rho[u'(\min\{q^*, z + \delta_h a\}) - 1]\}$. For all $z \geq z_2$, the marginal value of money is $\lambda[u'(\max\{z, \min\{z + \rho\delta_h a, \tilde{q}\}\}) - 1]$.

In all cases, the marginal value of money switches from $G_1(z)$ to $G_2(z)$ when z is sufficiently large, with the only difference being the thresholds. By following the argument in the proof of Proposition 7, it is straightforward to show that there exist $i_1 > i_2 \geq i_3 \geq i_4 > 0$ such that for all $i \geq i_1$, z solves $i = \lambda\{(1-\rho)[u'(z) - 1] + \rho[u'(\min\{q^*, z + \delta_h a\}) - 1]\}$. For all $i_2 \leq i < i_1$, $z = z_1$ where z_1 solves $i_1 = \lambda\{(1-\rho)[u'(z) - 1] + \rho[u'(\min\{q^*, z + \delta_h a\}) - 1]\}$. For all $i_3 \leq i < i_2$, z solves $i = \lambda[u'(z + \rho\delta_h a) - 1]$. For all $i_4 \leq i < i_3$, $z = z_2$ where z_2 solves $i_3 = \lambda[u'(z + \rho\delta_h a) - 1]$. Finally, for all $z \leq i_4$, z solves $i = \lambda[u'(z) - 1]$. $\square$

Proof of Proposition 10: I prove the comparative statics of z with respect to i. The rest of the comparative statics follows directly from Proposition 2. If $i \geq i_1'$, z solves $i = \lambda\{(1-\rho)[u'(z) - 1] + \rho[u'(\min\{q^*, z + \delta_h a\}) - 1]\}$. Then it is clear that z is decreasing in i. If $i_2' \leq i < i_1'$, $z = z_1$ where z_1 solves $i_1' = \lambda\{(1-\rho)[u'(z) - 1] + \rho[u'(\min\{q^*, z + \delta_h a\}) - 1]\}$, then z is unaffected by i. If $i_3' < i < i_2'$, z solves $i = \lambda[u'(z + \rho\delta_h a) - 1]$, then it is clear that z is decreasing in i. If $i_4' \leq i < i_3'$, $z = z_2$ where z_2 solves $i_3' = \lambda[u'(z + \rho\delta_h a) - 1]$, then z is unaffected by i. Finally, if $i \leq i_4'$, z solves $i = \lambda[u'(z) - 1]$, so z is decreasing in i. $\square$

Appendix C Endogenizing τ: Additional Results

In this section, we discuss Endogenizing τ when δ_l is assumed to be strictly positive. First, we consider the case where the cost κ must be paid after the realization of the consumption shock but before the realization of the quality shock. Let $\hat{a}$ denote the the asset holding of a consumer at the beginning of the AM. Let $B_1(z)$ denote the benefit of paying κ. We have several cases to discuss.
(1) Suppose that $z \geq z''$. We have

$$B_1(z) = \rho[u(\tilde{q}_h) + z + \delta_h\hat{a} - \tilde{q}_h] - \rho[u(q_h) + z + \delta_h\hat{a} - q_h], \tag{C.1}$$

where $\tilde{q}_h = \max\{z + \delta_h\hat{a}, q^*\}$, and $q^h = z + \delta_h a$ solves (B.49) when $s = 0$. It is straightforward to show that in this case, $B_1'(z) < 0$.
(2) Suppose that $z_s' \leq z_s < z_s''$. We have

$$\begin{aligned} B_1(z) =& \rho[u(\tilde{q}_h) + z + \delta_h\hat{a} - \tilde{q}_h] + (1-\rho)[u(\tilde{q}_l) + z + \delta_l\hat{a} - \tilde{q}_l] \\ &- \rho[u(q_h) + \delta_h(\hat{a} - a - s)] - (1-\rho)[u(q_l) + \delta_l(\hat{a} - a - s)], \end{aligned} \tag{C.2}$$

where $\tilde{q}_l = \max\{z + \delta_l\hat{a}, q^*\}$, $q_h = z + \bar{\delta}s + \delta_h a$, $q_l = \min\{z + \bar{\delta}s + \delta_l a, q^*\}$, and s and a solve (B.56) and (B.49). Recall that z does not affect q_h or q_l in this case. Hence, $B_1'(z) > 0$ if $\tilde{q}_h$ and/or $\tilde{q}_l$ are less than q^*, and $B_1'(z) = 0$ if otherwise.

(3) Suppose that $z_s < z_s'$. We have

$$B_1(z) = \rho[u(\tilde{q}_h) + z + \delta_h \hat{a} - \tilde{q}_h] + (1-\rho)[u(\tilde{q}_l) + z + \delta_l \hat{a} - \tilde{q}_l] - u(z + \bar{\delta}\hat{a}) \tag{C.3}$$

If $u'''(.) > 0$, then $B_1(z) > 0$ as long as $\tilde{q}_h < q^*$. Under the assumption that $a < \min\left\{\frac{\tilde{q}}{\delta}, \frac{q^*}{\delta_h}\right\}$, we have $q^* - \delta_h a - (\tilde{q} - \rho\delta_h a) = q^* - \delta_h a > 0$. Hence, $z + \delta_h a > q^*$ implies that $z + \rho\delta_h a > \tilde{q}$. In other words, $\tilde{q}_h < q^*$ when $z < z'$.

We summarize the equilibrium in the following proposition.

Proposition 11 *There exist $\mu^\dagger$ and $\mu^\ddagger$ such that*
(1) If $\mu > \mu^\ddagger$, consumers do not pay κ, and they pool in the DM.
(2) If $\mu^\dagger < \mu \leq \mu^\ddagger$, consumers pay κ.
(3) If $\mu \leq \mu^\dagger$, consumers do not pay κ, and they separate in the DM.

Now, we assume that consumers pay κ after they learn the quality of assets. In this case, only consumers with high-quality assets will pay the cost. Let $\tilde{\tau}$ denote the proportional of consumers who pay κ. Then, either $\tilde{\tau} = 1$ or $\tilde{\tau} = 0$. Intuitively, this is because the marginal value of real balances depends on whether consumers pay κ or not. Therefore, even if consumers are indifferent regarding paying κ or not ex post, they will strictly prefer different holdings of real balances ex ante, which means consumers will not be indifferent ex post, a contradiction. Now, let $\bar{B}_2(z)$ and $\underline{B}_2(z)$ denote the benefits of paying κ given $\tilde{\tau} = 1$ and $\tilde{\tau} = 0$, respectively. We have several cases.
(1) Suppose that $z \geq z''$. We have

$$\bar{B}_2(z) = \underline{B}_2(z) = u(\tilde{q}_h) + z + \delta_h \hat{a} - \tilde{q}_h - [u(q_h) + z + \delta_h \hat{a} - q_h], \tag{C.4}$$

where $\tilde{q}_h = \max\{z + \delta_h \hat{a}, q^*\}$, and $q_h = z + \delta_h a$ solves (B.49) when $s = 0$.
(2) Suppose that $z_s' \leq z_s < z_s''$. We have

$$\bar{B}_2(z) = u(\tilde{q}_h) + z + \delta_h \hat{a} - \tilde{q}_h - [u(q_h) + z + \delta_h \hat{a} - q_h], \tag{C.5}$$
$$\underline{B}_2(z) = u(\tilde{q}_h) + z + \delta_h \hat{a} - \tilde{q}_h - [u(\tilde{q}_h) + \delta_h(\hat{a} - a - s)], \tag{C.6}$$

where $\tilde{q}_h = z + \bar{\delta}s + \delta_h a$, and s and a solve (B.56) and (B.49). Recall that z does not affect $\tilde{q}_h$ in this case. When $\tilde{\tau} = 1$, if a consumer with high-quality assets chooses not to pay κ, then her optimal strategy is to offer a separating offer.
(3) Suppose that $z_s < z_s'$. We have

$$\bar{B}_2(z) = u(\tilde{q}_h) + z + \delta_h \hat{a} - \tilde{q}_h - [u(q_h) + z + \delta_h \hat{a} - q_h], \tag{C.7}$$
$$\underline{B}_2(z) = u(\tilde{q}_h) + z + \delta_h \hat{a} - \tilde{q}_h - u(z + \bar{\delta}\hat{a}). \tag{C.8}$$

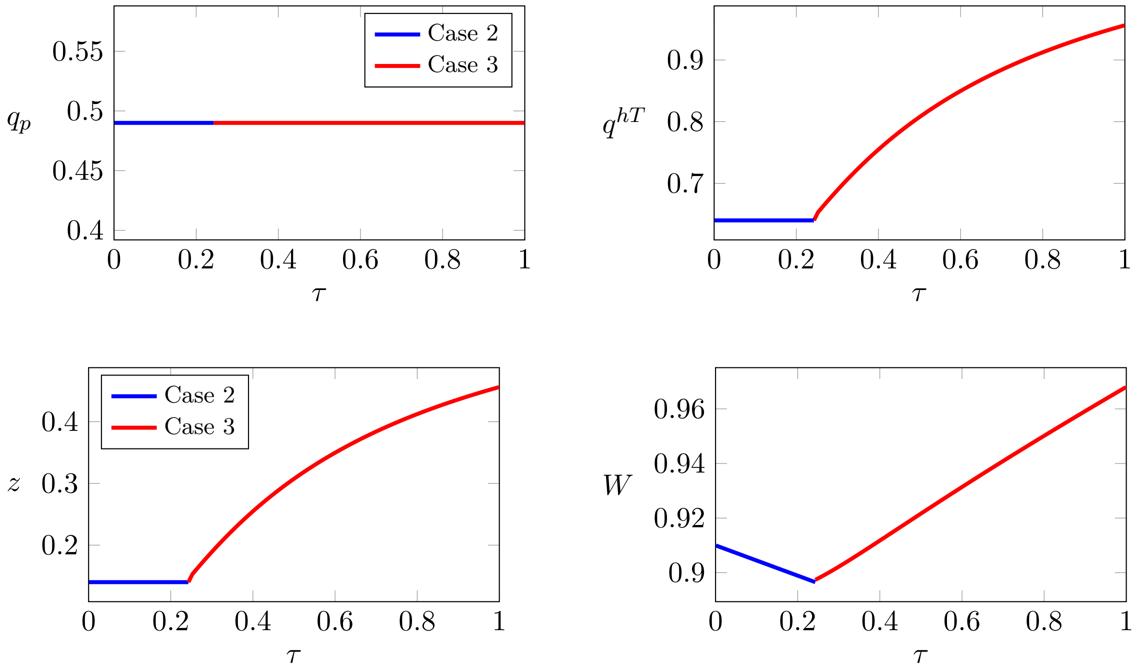
Similar to Case (2), when $\tilde{\tau} = 1$, if a consumer with high-quality assets chooses not to pay κ, then her optimal strategy is to offer a separating offer.

We summarize the equilibrium in the following proposition.

Proposition 12 *There exists $\mu^\diamond$ such that consumers with high-quality assets will pay κ if and only if $\mu \leq \mu^\diamond$.*

Appendix D Numerical Examples with an Exogenous τ

In this appendix, we provide some more numerical examples when τ is assumed to be exogenous (see Section 3). In the following examples, $u(q) = \frac{q^{1-\sigma}}{1-\sigma}$, $\sigma = 0.5$, $\lambda = 0.5$, $\rho = 0.7$, $\tau = 0.5$, $a = 0.5$, $\delta_h = 1$, and $i = 0.08$.



In the following example, $i = 0.25$.