Web Appendix: Monetary Policy, Asset Prices, and Liquidity in Over-the-Counter Markets

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WA.1 Comparative statics with respect to OTC market conditions

We begin with λ, the bargaining power of the C-type, and work with the first-order conditions of the buyers' optimal portfolio choice described in Appendix A.1.2 of the paper. An increase in λ will decrease money demand in Regions 1, 3, and 5 (by decreasing the gradient J_1).[1] However, equilibrium real balances will increase in λ if equilibrium is on the boundary of Regions 3 and 5. We obtain this counterintuitive result because on that boundary, the objective function has a kink and neither first-order condition applies. Instead, equilibrium real balances are determined by the condition that we have to be on this boundary, which means that the trading value of the real balances held by an N-type has to equal the trading value of the assets held by a C-type. For higher λ, the amount of real balances equal in trading value to a fixed quantity of assets has to rise.

Everywhere else, the first-order conditions do apply and λ has an unambiguous negative effect on real balances. As a further consequence, the cutoffs $\tilde{\mu}$, μ', μ'', and μ''' (see Figure 2 in the paper) unambiguously decrease with λ for a given A.

The effects of λ on q_2 (the amount of DM trade by those who did rebalance) and on equilibrium welfare (measured by average trade in the DM) are the same as the effect on real balances: decreasing in the interior of all regions because buyers take liquidity for granted if λ is high, but increasing on the boundary of Regions 3 and 5.

The CM asset price ψ in Region 5 is described by equation 13 in the paper. It depends positively on λ and negatively on q_2, so the overall effect of λ must be positive in this region. However, if a small increase in λ were to push equilibrium into the boundary region, then ψ would become indeterminate, and a further increase in λ could push equilibrium into Region 3, in which case ψ would "fall" to the fundamental value βd.

The OTC asset price $\varphi\psi_I$, measured in real terms at the preceding CM, is described in equation 14.[2] Like ψ, it depends positively on λ directly. But there is also an indirect effect through the ratio $[u(q_2) - u(q_1)]/[q_2 - q_1]$, which is strictly decreasing in real balances and therefore increasing in λ. This indirect effect could in principle dominate the direct effect although it does

[1] These regions are defined in terms of portfolios of real balances and asset holdings, which is the domain of the buyer's objective function, and depicted in Figure 1 in the paper.

[2] We could choose to deflate the nominal OTC price with the price level in the subsequent CM instead, which would change both the interpretation and the analysis of this object.

not do so for any of our numerical examples.

It is worth noting that because the inflation cutoffs decrease in λ, the size of Regions 1 and 5 is maximized for $\lambda = 0$. So a low value of λ, while driving the liquidity premium of the asset price in the CM to zero, is not at all equivalent to shutting down the OTC market, and suggests caution in interpreting the size of a theoretical (or empirical) liquidity premium as a measure of the welfare contribution of OTC rebalancing in an economy.

We next turn to f, the probability of matching in the OTC market. Again inspecting the money demand equations, we see that real balances decrease with f in Regions 1 and 5, because if matching in the OTC becomes more likely, money becomes easier to obtain *ex post* and buyers will value it less *ex ante*. Consequently, the cutoffs $\tilde{\mu}$, μ', and μ'' are decreasing in f. However, the effect of f is ambiguous for high inflation: for high λ, real balances and the cutoff μ''' are also decreasing in f, but for low λ, the opposite happens. The threshold level of λ depends on the curvature of the utility function u.

As f, unlike λ, does not affect OTC trade once the portfolios are determined, it does not affect the level of real balances which defines the boundary of Regions 3 and 5. It has therefore no effect on q_1, q_2, or $\varphi\psi_I$ in equilibria on this boundary, although an equilibrium will leave the boundary if f changes by a large amount and one of the first-order conditions becomes binding.

Increasing f has a large and positive direct effect on welfare, because it enables more C-types to match in the OTC market and bring more real balances into the DM. However, buyers take some of this benefit for granted, which tends to reduce both q_1 and q_2 indirectly. In theory, this could make f have a negative impact on welfare but we have not been able to find parameters or functional forms for which this is the case in our model.

The CM asset price ψ is equal to the fundamental in Regions 1 and 3 and is therefore not affected by f. In Region 5, f has a positive direct effect on ψ and also a positive indirect effect, as ψ is decreasing in q_2, which is in turn decreasing in f in this region. On the boundary of Regions 3 and 5, ψ is indeterminate but the range of indeterminacy is increasing in f, and the range $[\mu'', \mu''']$ of inflation rates for which equilibrium is in the boundary region will expand in f if λ is small.

Finally, the OTC asset price, measured in real terms as before, is not directly affected by f, so the only effect is the indirect one through $[u(q_2) - u(q_1)]/[q_2 - q_1]$. As this ratio is strictly decreasing in real balances, it is increasing in f except possibly in Region 3, and except in the boundary region where real balances are unaffected by f. In conclusion, $\varphi\psi_I$ is decreasing in f for low inflation (in Regions 1 and 5) but ambiguous in general.

WA.2 A version of the model with a competitive secondary asset market

Assume that every agent can enter a secondary market in which assets are traded for money after the liquidity shock occurs. With rational expectations, at the beginning of a period, agents take as given the primary market prices φ (fruit per unit of money) and ψ (fruit per unit of asset), the *competitive* secondary market price ψ_C (money per unit of asset), and the future primary market price $\hat{\varphi}$. Given these prices, they specify demands $\hat{m}$ and $\hat{a}$ in the primary market, and asset demands $-\chi^C$ (if a C-type) and χ^N (if an N-type). Because there is no aggregate uncertainty, total asset supply in the secondary market will be $\ell\chi^C$ and total asset demand will be $(1-\ell)\chi^N$.

The objective function becomes:

$$\begin{aligned} J(\hat{m},\hat{a},\chi^C,\chi^N) = & -\varphi\hat{m} - \psi\hat{a} \\ & + \ell\left\{u\left[\beta\hat{\varphi}(\hat{m}+\psi_C\chi^C)\right] + \beta(\hat{a}-\chi^C)\right\} \\ & + (1-\ell)\left[\beta\hat{\varphi}(\hat{m}-\psi_C\chi^N) + \beta d(\hat{a}+\chi^N)\right] \\ \text{subject to } & \chi^C \le \hat{a} \text{ and } \hat{m} \ge \psi_C\chi^N, \end{aligned}$$

and with Lagrange multipliers θ_1 and θ_2, the representative agent's problem at the beginning of the period becomes:

$$\max_{\{\hat{m},\hat{a},\chi^C,\chi^N,\theta_1,\theta_2\}} J(\hat{m},\hat{a},\chi^C,\chi^N) + \theta_1\left(\hat{a}-\chi^C\right) + \theta_2\left(\hat{m}-\psi_C\chi^N\right).$$

Writing $q \equiv \beta\hat{\varphi}(\hat{m}+\psi_C\chi^C)$, we obtain the following first-order conditions:

$$\begin{aligned} \hat{m}: \quad & 0 = -\varphi + \beta\hat{\varphi}\left[\ell u'(q) + 1 - \ell\right] + \theta_2 \\ \hat{a}: \quad & 0 = -\psi + \beta d + \theta_1 \\ \chi^C: \quad & 0 = \ell\left[\beta\hat{\varphi}\psi_C u'(q) - \beta d\right] - \theta_1 \\ \chi^N: \quad & 0 = (1-\ell)\left[\beta d - \beta\hat{\varphi}\psi_C\right] - \psi_C\theta_2, \end{aligned}$$

together with the usual complementary slackness conditions. The market clearing conditions are $\hat{a} = A$, and $\ell\chi^C = (1-\ell)\chi^N$. Stationarity requires $\varphi = (1+\mu)\hat{\varphi}$.

Before we describe equilibrium, note that adding up the FOCs for $\hat{m}$ (multiplied by ψ_C) and χ^N and subtracting the FOCs for $\hat{a}$ and χ^C yields $\psi = \varphi\psi_C$: The real asset price in the primary market must equal the real price in the secondary market, and there are no arbitrage opportunities for any outside parties.

Equilibrium: Taking the third and fourth FOC together, we can see that either $q = q^*$ or one of the constraints must bind: the Lagrange multipliers must be non-negative, and if $u'(q) = 1$, this forces both of them to be zero. But by the first FOC, we can see that $q = q^*$ and $\theta_2 = 0$ are possible only if $\mu = \beta - 1$. Consequently, $q = q^*$ is attainable only at the Friedman rule, and correspondingly, $\hat{m} + \psi_C \chi^C < m^*$ for any $\mu > \beta - 1$. So focusing on $\mu > \beta - 1$, we are left with three regions of the solution. Region A is intended to loosely correspond with Region 5 of the OTC model, Region B with the aggregate boundary of regions 3 and 5, and Region C with Region 3. Notice that, with a frictionless secondary market, there is no equivalent to Region 1. This will be essential for our welfare comparison between this model and the baseline model with OTC secondary asset trade.

Region A: Let $\theta_1 > 0$ and $\theta_2 = 0$. We get $\psi_C = d/\hat{\varphi}$, $(1+\mu)/\beta = 1-\ell+\ell u'(q)$, and $\chi^C = \hat{a} = A$. Because constraint 1 binds, we have $q = \beta z + \beta d A$, where $z = \varphi M = \hat{\varphi}\hat{m}$ denotes real balances.

Region B: Let $\theta_1 > 0$ and $\theta_2 > 0$. We get $\psi_C = \hat{m}/\chi^N$, and real balances solve the equation:

$$\left[\frac{1+\mu}{\beta} - \ell u'\left(\frac{\beta z}{\ell}\right)\right] z = \ell d A$$

Region C: Let $\theta_1 = 0$ and $\theta_2 > 0$. We get $\psi_C = \beta d/[(1+\mu)\hat{\varphi}]$, $(1+\mu)/\beta = u'(q)$, and $\chi^C = [(1-\ell)/\ell][(1+\mu)/\beta]z/d$. Because constraint 1 is slack, $\chi^C < \hat{a}$, which implies $[(1-\ell)/\ell](1+\mu)z < \beta d A$ in the aggregate. Because constraint 2 binds, we have $q = \beta z/\ell$.

Taking these three regions together, we can define cutoffs μ^I and μ^{II} such that $\mu \in (\beta-1, \mu^I)$ puts the solution in Region A, $\mu \in (\mu^I, \mu^{II})$ puts the solution in Region B, and $\mu \in (\mu^{II}, \infty)$ puts the solution in Region C. However, μ^{II} is only unique if $u'(q)q$ is non-decreasing in q. With this additional assumption, the cutoffs solve:

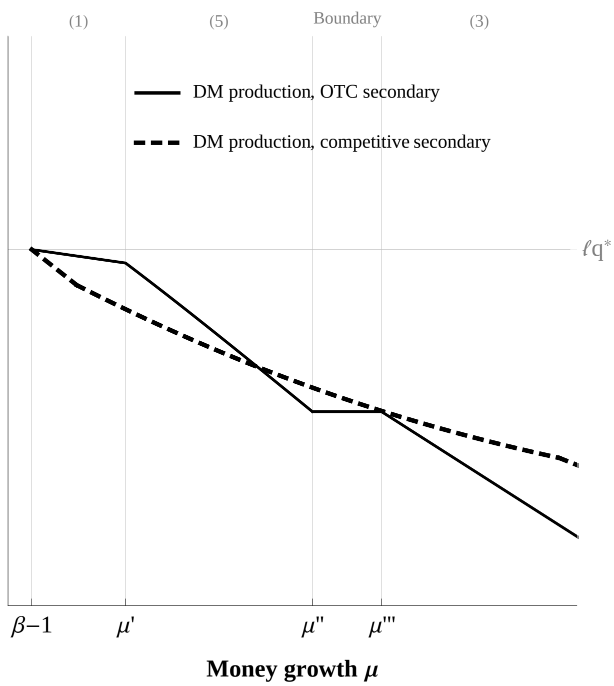
$$\frac{1+\mu^I}{\beta} = 1 - \ell + \ell u'\left(\frac{\beta d A}{1-\ell}\right)$$
$$\frac{1+\mu^{II}}{\beta} = u'\left(\frac{\beta d A}{1-\ell}\frac{\beta}{1+\mu^{II}}\right)$$

Define $\bar{A}_C = \frac{1-\ell}{\beta d} q^*$. It is straightforward to show that $\beta - 1 < \mu^I < \mu^{II}$ if $A < \bar{A}_C$, and that both μ^I and μ^{II} are strictly decreasing in A (here, we again need to use the assumption that $u'(q)q$ is non-decreasing in q). If $A \geq \bar{A}_C$, the solution is always in Region C.

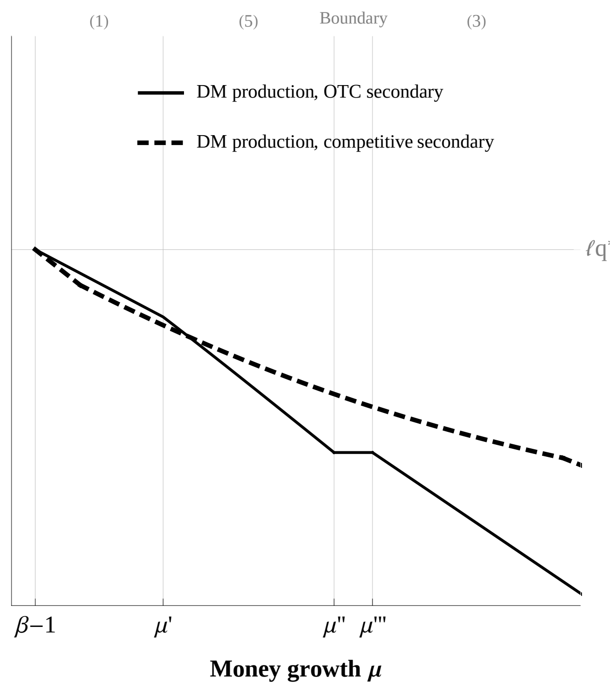
The behavior of the system is easy to describe. Real balances z are a strictly decreasing function of inflation, and so is aggregate LW production ℓq. However, z tends to decrease as a function of A, while ℓq tends to increase (unless $A > \bar{A}_C$). The real asset price ψ is the same in the primary and secondary market. As a function of inflation, it is continuous; furthermore, it increases strictly in Region A, decreases to the fundamental value in Region B (not necessarily monotonically), and is at the fundamental value in Region C.

OTC versus competitive secondary asset trade: a numerical example

In the paper, we show that if inflation is sufficiently low, welfare will be higher in the model where the secondary asset market is OTC (Proposition 4). Since in the model with a competitive secondary market all C-types get a chance to trade in that market, in order to make the two models comparable, we assumed that in the OTC market $f = \ell$ (which requires $\ell \leq 1/2$). Below we provide examples showing that the bargaining friction is so powerful that even if there are severe search frictions in the OTC ($f << \ell$), equilibrium welfare can still be higher in the model with an OTC secondary asset market.



(a) $f = 0.9\ell$



(b) $f = 0.5\ell$

Figure 1: Comparison of q_{DM} in the two models for $f = 0.9\ell$ and $f = 0.5\ell$. The region cutoffs μ^I and μ^{II} of the competitive model are not shown but can be detected as slight kinks in the dashed lines.