A Tractable Model of Indirect Asset Liquidity*

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Abstract: Assets have "indirect liquidity" if they cannot be used as media of exchange, but can be traded to obtain a medium of exchange (money) and thereby inherit monetary properties. This essay describes a simple dynamic model of indirect asset liquidity, provides closed form solutions for real and nominal assets, and discusses properties of the solutions. Some of these are standard: assets and money are imperfect substitutes, asset demand curves slope down, and money is not always neutral. Other properties are more surprising: prices are flexible but appear sticky, and an increase in the supply of indirectly liquid assets can decrease welfare. Because of its simplicity, the model can be useful as a building block inside a larger model, and for teaching concepts from monetary theory.

Keywords: monetary-search models, asset liquidity, asset prices, monetary policy, monetary aggregates

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Recently, interest has increased in tractable models of asset liquidity that are easy to analyze and can provide benchmarks for understanding the relationship between monetary policy and asset market frictions. For example, Williamson (2012), Andolfatto and Williamson (2015), and Rocheteau, Wright, and Xiao (2014) analyze policy using models in which multiple assets can take the role of a medium of exchange, usually to different degrees, which makes them imperfect substitutes and the question of their relative supplies interesting. Furthermore, a number of recent papers have suggested that the notion of asset liquidity may be the key to rationalizing some long-standing asset pricing-related puzzles (see for example Lagos, 2010, Geromichalos, Herrenbrueck, and Salyer, 2016, and Jung and Lee, 2015).

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The assumption that assets such as US Treasuries or claims to capital serve as media of exchange is popular in monetary theory (going back to Geromichalos, Licari, and Suarez-Lledo, 2007, and Lagos and Rocheteau, 2008), but is sometimes questioned based on realism. In Geromichalos and Herrenbrueck (2016a), we demonstrated that *indirect* asset liquidity can support many of the same conclusions, while offering an alternative microfoundation of imperfect asset substitutability: in this conception of liquidity, assets are substitutes to money because agents can sell them in a secondary financial market, and thus obtain the money needed to make a purchase. This detail is important because it implies that the structure of financial markets has first-order consequences for the 'moneyness' of different types of assets, and therefore their trading volumes and prices. It is also empirically supported, as we discuss in Geromichalos et al. (2016).

The goal of this essay is to present a very simple model which preserves the essential concept of 'indirect liquidity', but which is maximally tractable. This allow us to illustrate important results which are latent in some earlier papers, but perhaps not fully developed or understood. As a first example, we show that the optimal supply of indirectly liquid assets is not-too-large and may be zero. Second, the ease at which assets can be traded affects the price level. The reason is that a fall in ease of trading reduces the effective supply of liquidity and thereby increases, in textbook terms, the demand for money. Third, our model implies a new formula for constructing a monetary aggregate that contrasts with popular alternatives. Finally, we think the simple model will be useful as a building block for general applications in academic and policy work.

1 The model

As in the framework of Lagos and Wright (2005), time is discrete and goes on forever, and agents discount the future at rate $\beta < 1$. Each period is divided into three stages. At the end of a period, a competitive "centralized" market opens in which agents produce and consume a general consumption good over which they have linear preferences, and allocate their wealth between two different assets. The first asset, "money" M, is fully liquid in the sense that it can be used to purchase special consumption goods. These goods are traded in a "decentralized" goods market which opens in the middle of each period; this market is subject to search frictions, and agents are anonymous and lack commitment. Therefore, a medium of exchange is required, and money is the only asset that can play this role. The second asset is a real discount bond b which is illiquid in the sense that it cannot be used to purchase consumption. Instead, these bonds can be *liquidated* in an over-the-counter asset market (OTC henceforth), which opens at the beginning of each period, for money; because agents anticipate this, they may value the bonds at a liquidity premium in the centralized market even though the bonds are never used as media of exchange. In this sense, bond liquidity is *indirect*.

There are two distinct types of agents: consumers and producers of special goods. The latter are trivial in this model: as they can never consume special goods, they have no need for liquid assets. There is a measure ℓ of them, their cost of production is linear with slope normalized to 1 in terms of the general consumption good, and they have zero bargaining power in the decentralized market. Because the consumer-agents make all the interesting economic decisions, we simply refer to them as "agents" from here on. These agents value consumption of q units of the special consumption good at $u(q)$, a strictly increasing and concave function. There is a measure 1 of them, and when trading with a producer, they make a take-it-or-leave-it offer that exactly compensates for the cost of production, so the total surplus $u(q) - q$ in every trade for the special good is also the consumer's surplus. There is a quantity q^* which satisfies $u'(q^*) = 1$; we call it the "first-best" quantity because it maximizes the total surplus. Because a medium of exchange is required, however, actual trade in the special good will often be for less than this quantity.

There is a government whose only function is to maintain the supply of money and bonds. Bonds are issued and redeemed in the centralized market, and new money is introduced via lump-sum transfers to agents in the centralized market.[1] The money supply grows at rate $\mu > \beta - 1$, which determines inflation in steady state; for convenience, we express this with the parameter $i \equiv (1 + \mu - \beta)/\beta$.[2]

[1] Because of linear preferences in this market, it does not matter for aggregate outcomes who initially receives the new assets. In every equilibrium all agents will end the period with the representative portfolio (or be indifferent to doing so). We also do not explore the motivation of the government to choose particular levels of transfers or debt. In a broader context, whether monetary policy is set in terms of a monetary or in terms a fiscal transfer can matter a great deal, especially in a liquidity trap (Herrenbrueck, 2014; Andolfatto and Williamson, 2015).

[2] This parameter can be interpreted as the cost of liquidity. It is sometimes called *the* nominal interest rate; in a world where liquidity premia exist, it should be clarified that i would be the

At the beginning of every period, a proportion $\ell < 1$ of agents learn that they will be matched with a producer, and thereby have the opportunity to consume the special good in this period. However, because at the end of the last period they did not know if the consumption opportunity would arise today, all agents hold the same amounts of money and bonds; in particular, some hold money that they do not need in the same period, and others hold bonds that they would like to sell in order to get more money.

This portfolio reallocation happens in an OTC market characterized by frictions.[3,4] The first friction is that trade is bilateral and matching is random, and the total number of matches is $f < \min\{\ell, 1-\ell\}$ so that nobody matches with certainty. Second, to make things tractable, we assume that the asset seller makes a take-it-or-leave-it offer and therefore extracts the full surplus from the OTC trade. Compared to more general bargaining protocols, this simplification matters qualitatively only for the OTC market price of bonds, although it will quantitatively affect other variables. It also eliminates an asset pricing indeterminacy present in Geromichalos and Herrenbrueck (2016a).

An equilibrium will be characterized by three endogenous variables. The first is the quantity of special goods bought by consumers who did not obtain additional money in the OTC market, denoted by q_0. The second is the quantity of special goods bought by consumers who *did* obtain additional money in the OTC market, denoted by $q_1 > q_0$. The third equilibrium object is the real price of bonds in the centralized market, denoted by ψ.

To make things even simpler, we want to restrict attention to equilibria in which asset buyers are never constrained by their money holdings in the OTC. A sufficient condition which assures this is that inflation is not too large, in particular: $i < (\ell - f)\left[u'\left(q^*/2\right) - 1\right]$. We will maintain this assumption from now on.

We might also be interested in characterizing welfare, which is here given by the surplus of consuming special goods, averaged between consumers of special goods who had the opportunity to rebalance and those who did not:

nominal interest rate on an asset that is either perfectly illiquid or in such large supply so that its liquidity properties do not matter at the margin.

[3] Other models of portfolio reallocation include Alvarez, Atkeson, and Kehoe (2002), Berentsen, Camera, and Waller (2007), Berentsen, Huber, and Marchesiani (2014), and Lagos and Zhang (2015).

[4] OTC market structure is empirically relevant for a large class of assets, including some that we think of as very liquid (Duffie, Gârleanu, and Pedersen, 2005; Afonso and Lagos, 2015).

$$\mathcal{W} = (\ell - f)\,[u(q_0) - q_0] + f\,[u(q_1) - q_1]$$

1.1 Equilibrium with real bonds

We can now state three equations determining general equilibrium in the endogenous variables $\{q_0, q_1, \psi\}$.

$$\begin{aligned} &\text{(Money demand)} && i = (\ell - f)\,[u'(q_0) - 1] + f\,[u'(q_1) - 1] \\ &\text{(OTC trade)} && q_1 = \min\{q^*, q_0 + b\} \\ &\text{(Bond demand)} && \psi = \beta\,[1 + f\,(u'(q_1) - 1)] \end{aligned}$$

In this environment, the Friedman Rule $i = 0$ implements the first-best allocation $q_0 = q_1 = q^*$. In what follows, we analyze welfare outcomes conditional on a fixed $i > 0$, and we are agnostic as to why a particular i was chosen by the policy maker.

The formula for the bond price ψ can be interpreted as the product of the "fundamental value" β and a liquidity premium. (In particular, it can easily imply a negative real interest rate if i is large enough.) Comparative statics with respect to bond supply or bond liquidity are now straightforward. First, observe that if $b \geq q^* - (u')^{-1}[1 + i/(\ell - f)]$, then an OTC bond seller will not sell all of her bonds. In this case, the bond supply is "abundant", and it does not affect the equilibrium quantities, prices, or welfare at the margin. But bond liquidity does matter, even if the bond supply is abundant: an increase in f will make it easier for agents to obtain cash *ex post*, and they will therefore demand less *ex ante*. This will drive down real balances (which may matter on its own, as an increase in the price level), and for a large enough increase in f, real balances will be so low that bonds are not abundant after all.

This other case, where an OTC bond seller will sell all of her bonds and the bond supply is therefore "scarce", is obtained when $b < q^* - (u')^{-1}[1 + i/(\ell - f)]$. It is much more interesting: increases in the bond supply b reduce real balances pre-trade (q_0) but increase them post-trade (q_1), and therefore also increase the bond yield $1/\psi - 1$; the aggregate bond demand curve is downward sloping.

In the scarce case, the effect of the bond supply on welfare is more complex and depends on the shape of the utility function:

$$\frac{d\mathcal{W}}{db} = f(\ell - f)\,\frac{[u'(q_1) - 1] \cdot u''(q_0) - [u'(q_0) - 1] \cdot u''(q_1)}{(\ell - f)\,u''(q_0) + f\,u''(q_1)}$$

In general, the sign of the derivative is ambiguous, but we can say more in two special cases. First, if the bond supply approaches the abundant level, we have $q_1 \to q^*$ (by definition of "abundant") and therefore $u'(q_1) \to 1$. As long as the reasonable assumption $u''(q^*) < 0$ is satisfied, this implies that $d\mathcal{W}/db$ is always negative when b approaches the abundant level.[5] Second, because u is concave, $u'(q_0)$ is always greater than $u'(q_1)$. If $u''(q_1)$ is greater in absolute value than $u''(q_0)$ (both are of course negative, so $u'''(q) \leq 0$ would be a sufficient condition for that), then welfare is always *decreasing* in the real supply of bonds – an outcome in contrast to most other models of bond liquidity.

We now turn our attention to a quadratic utility function for which we can compute general equilibrium explicitly. Let $u(q) \equiv (1+\sigma)q - \frac{1}{2}q^2$, which has the properties $u'(q) - 1 = \sigma - q$ and $q^* = \sigma$. First, if $b \geq i/(\ell - f)$, then the bond supply is abundant and does not affect the equilibrium:

$$q_0 = \sigma - \frac{i}{\ell - f} \tag{1a}$$

$$q_1 = \sigma \tag{1b}$$

$$\psi = \beta \tag{1c}$$

If, on the other hand, $b < i/(\ell - f)$, then the bond supply is scarce and the equilibrium is:

$$q_0 = \sigma - \frac{i}{\ell} - \frac{f}{\ell}b \tag{2a}$$

$$q_1 = \sigma - \frac{i}{\ell} + \frac{\ell - f}{\ell}b \tag{2b}$$

$$\psi = \beta\left[1 + \frac{f}{\ell}\Big(i - (\ell - f)\, b\Big)\right] \tag{2c}$$

This formula for the bond price ψ has some very convenient properties. The liquidity premium is linear in the cost of carrying money (i) and in the aggregate supply of bonds (b). It is also strictly increasing (although not linear) in the probability of trade in the OTC (f) for the entire range $b \in [0, i/(\ell - f))$ where the

[5] This result is not sensitive to parameters or the OTC bargaining protocol; it is just an application of the envelope theorem to the fact that $q_1 \to q^*$ for a large bond supply. Welfare at low bond supplies is another matter. In a similar framework, Huber and Kim (2015) show that the optimal bond supply is always intermediate – between zero and the abundant level – as long as bargaining power in the OTC market is also intermediate.

liquidity premium is positive, and this range is itself expanding in i and f.

Perhaps surprisingly, the bond price is independent of σ (which can be interpreted as the *size* of the liquidity shocks). The intuition is that the liquidity premium measures the ability of the asset to catalyze a need to obtain liquidity *ex post*, which arises because agents hold too little liquidity *ex ante*. If σ increases, liquidity becomes more valuable both *ex post* and *ex ante*, so that the value of rebalancing is exactly unchanged in this special case.

As the quadratic utility function satisfies $u'''(q) = 0$, it implies the counterintuitive result that welfare is decreasing in the real supply of bonds throughout, as long as they are scarce. Using $u''(q) = -1$, we can compute directly:

$$\frac{d\mathcal{W}}{db} = -\frac{f(\ell - f)}{\ell}\, b$$

How can a greater supply of liquid bonds be bad for welfare? The reason is simply that as agents anticipate that it has become easier to obtain money *ex post*, they hold less money *ex ante* to self-insure against the liquidity shocks. Consequently, q_1 increases (up to q^*) while q_0 decreases. The key thing to notice is that if the bond supply is large enough, then $q_1 \approx q^*$, so the surplus of an agent who did rebalance in the OTC is almost flat. By the envelope theorem, this agent's marginal increase in utility is negligible. But as $q_0 < q_1$, the decrease in q_0 that results from a larger bond supply gets a large negative welfare weight.

Total consumption could in principle go in both directions, but in the special case of quadratic utility, it is easy to compute that it equals $\ell\sigma - i$ independently of the bond supply. As a consequence, total welfare must be decreasing in the bond supply because the negative effect on q_0 always has a larger marginal utility weight than the positive effect on q_1.

Our analysis has a flavor of the famous result that welfare may be higher in an economy where bonds are illiquid in some sense (Kocherlakota, 2003; also, Geromichalos and Herrenbrueck, 2016a, and Berentsen et al., 2014).[6] However, our focus here is on changes in the supply of bonds for constant levels of liquidity, and our take-away message is twofold. First, the aggregate supply of liquidity is

[6] Certainly, we can compute the comparative statics of f as well. In the special case with quadratic utility, average consumption is independent of f but welfare is U-shaped: maximized at the corners $f = 0$ and $f = \ell$ and minimized in the interior. With real bonds, the welfare-minimizer is $f = 0.5\ell$, and with nominal bonds (case below), it is decreasing in the bonds-to-money ratio.

not a sufficient statistic for welfare; the distribution of liquidity matters. Second, the optimal supply of bonds does not drive liquidity premia to zero.

1.2 Equilibrium with nominal bonds

Instead of a real bond in supply b, say that there is a nominal bond in supply B. In this case, the real bond price is not very meaningful, so we want to use the nominal bond price p instead. (Using the money demand equation, it is easy to show that $p \leq 1$, i.e. the nominal yield on this bond cannot be negative even though the real one can.) The equations governing general equilibrium are as follows:

$$\text{(Money demand)} \qquad i = (\ell - f)\left[u'(q_0) - 1\right] + f\left[u'(q_1) - 1\right]$$

$$\text{(OTC trade)} \qquad q_1 = \min\left\{q^*, q_0\left(1 + \frac{B}{M}\right)\right\}$$

$$\text{(Bond demand)} \qquad p = \frac{1}{1+i}\left[1 + f\left(u'(q_1) - 1\right)\right]$$

In an important contrast to the previous model, money is no longer neutral, at least in the case where bonds are scarce. We will discuss this in detail below in Section 2. But first, we continue our quest for closed-form solutions and use the quadratic utility function from Section 1.1 again. If $B/M \geq i/[(\ell - f)\sigma - i]$, then the bond supply is abundant and the equilibrium is:

$$q_0 = \sigma - \frac{i}{\ell - f} \tag{3a}$$

$$q_1 = \sigma \tag{3b}$$

$$p = \frac{1}{1+i} \tag{3c}$$

If, on the other hand, $B/M < i/[(\ell - f)\sigma - i]$, then the bond supply is scarce and the equilibrium is:

$$q_0 = \frac{M}{\ell M + fB}(\ell\sigma - i) \tag{4a}$$

$$q_1 = \frac{M+B}{\ell M + fB}(\ell\sigma - i) \tag{4b}$$

$$p = \frac{1}{1+i}\left(1 + f\,\frac{i\,(M+B) - (\ell - f)\,\sigma\,B}{\ell M + fB}\right) \tag{4c}$$

The resulting equations are more complex than Equations (2); for example, the bond price is no longer linear in the bond supply. However, the main comparative statics are comparable. An increase in the bonds-money ratio B/M reduces the bond price p, increases q_1 but reduces q_0 and, thereby, total welfare – up to the point where $B/M \geq i/[(\ell - f)\sigma - i]$, after which further increases have no effect.

We also gain some new insights from this example. For one, in contrast to the example with real bonds, the bond price is decreasing in σ, the size of the liquidity shocks. Intuitively, an increase in the overall demand for liquidity now increases the demand for money (which is weighted by ℓ) by more than the demand for bonds (which is weighted by $f < \ell$); this drives down the bond price, potentially to a point where bonds are no longer scarce and their price is at the fundamental level. The specific outcome depends on the shape of the utility function, but it is a powerful example which shows that how exactly we model the "demand for liquidity" matters in subtle ways.[7]

2 Discussion

In this essay, we have constructed a simple model of indirect asset liquidity that admits convenient closed-form solutions for a variety of applications. In a model with real bonds, the resulting equilibrium equations are exceptionally simple: for example, the price of real bonds is linear in the bond supply and in the cost of liquidity. In a model with nominal bonds, the equations become less clean, but clearly illustrate the non-neutrality of money. To analyze this point in more detail, consider the determination of the general price level, defined as the ratio of money supply to real money demand, $P \equiv M/q_0$. Assuming that the bonds are scarce in OTC trade, we can transform Equation (4a) from the quadratic model to obtain:

$$P = \frac{\ell M + fB}{\ell\sigma - i} \tag{5}$$

As the equation reveals, the price level is increasing as a function of inflation,

[7] There is also an interesting technical difference compared with the model of real bonds: the equation describing how the post-trade quantity q_1 is determined from the pre-trade quantity q_0 is now multiplicative instead of additive. This will only interest modelers, but it makes it possible to get closed-form solutions with a CRRA utility function with general risk aversion. The resulting equations can be found in the appendix below.

decreasing as a function of the size or frequency of the liquidity shocks, and increasing in the liquidity of asset markets. This result is important: in particular, it stands to reason that a collapse of financial intermediation such as the one seen in 2008/09 would affect the economy as a simultaneous decrease in the broad supply of liquidity services ($\ell M + fB$), and an increase in the demand for narrow ones (M in this model). If monetary policy was unchanged, this should have caused a fall in the price level; equivalently, since we saw a big increase in the monetary base, the argument explains why this increase has not been inflationary.

Notably, of course, the example shows that money is not neutral when nominal bonds are scarce: an increase in M will cause an increase in p and q_0, and a decrease in q_1. Why? The reason is that the price level is *proportional to the total stock of nominal assets that have a liquidity role*, not to the money supply alone. When only one of these assets increases in quantity, then the price level will not increase in proportion. Changing the money supply via typical open market operations, where money is exchanged for another fairly liquid asset category such as government bonds or bank reserves, will only amplify the non-neutrality (see also Rocheteau et al., 2014). An increase in M financed by a decrease in B will increase the bond price (i.e. reduce nominal interest rates on the bond); as a result, pass-through of the monetary shock to the price level will be incomplete. Even though prices are flexible in this model, they can *look* sticky.

In contrast to some of the models listed at the beginning of this essay, the simple model here is not capable of exhibiting a liquidity trap situation where bond demand becomes flat at low interest rates. However, this is not true of the concept of indirect liquidity in general: Herrenbrueck (2014) describes a continuous-time model of indirect liquidity in which a liquidity trap can exist.

The idea that many assets contribute to providing monetary services is not new; for example, Fried and Howitt (1983) suggested that future research should establish "the appropriate definition of money for questions of inflation and unemployment". One popular approach is the Divisia method for constructing monetary aggregates (Barnett, 1978, 1980). This method involves first determining what the fundamental cost of liquidity is (i in our model), then weighting assets according to their "user cost". For example, if it happens that $i = 6\%$, then an asset yielding 3% receives half the weight of cash (which yields 0%) in the monetary aggregate. Our model identifies a problem with this method. In Equation (5), the weights on money and bonds can be interpreted as their *average* contributions to the supply of liquidity, while the difference between their yields and the fundamental yield i measures their *marginal* contributions. This matters both for the level and for changes. For one example, the liquidity premium on an asset can be zero if the asset is illiquid, but also if it is abundant (and it may be very liquid). Either way, it will receive zero weight in the Divisia aggregate. For another example, an increase in the bond supply B will increase the bond yield and thus reduce the weight on bonds in the Divisia aggregate, leading it to underestimate the increase in the overall liquidity supply.[8]

Despite its resemblance to the undergraduate textbook model of money demand, this model has more subtle implications. For one, while the demand for liquid assets certainly depends on the total demand for goods, it is not necessarily true that different levels of income are "associated" with different interest rates, even for a fixed combination of asset supplies. As the solutions above make clear, bond yields may depend on the overall demand for liquidity, but not necessarily – because a change in this demand may be distributed on all assets in proportion to their existing uses, or it may not be. The second difference from the simple textbook model is that changes in the interest rate (real or nominal) on a particular liquid asset do not reveal much about the interest rate available to anyone not issuing this asset, or even about i, a point which has of course been recognized for a long time. However, questions of this kind are exactly what our simple model of asset liquidity can usefully address.[9]

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[8] Li, Rocheteau, and Weill (2012) also study the weights of partially liquid assets in a monetary aggregate, and emphasize the distinction between the private value and the social value of an asset.

[9] For example, Geromichalos and Herrenbrueck (2016b) use it to study the strategic interaction of asset issuers when multiple assets can have this indirect liquidity role.

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APPENDIX – NOT FOR PUBLICATION

As suggested earlier, we can also get closed form solutions for the model with nominal bonds if we use a CRRA (power) utility function, which we could not do in the model with real bonds. Let $u(q) = q^{1-\gamma}\sigma^{\gamma}/(1-\gamma)$, with $\gamma > 0$, which again has the property that $q^* = \sigma$. Bonds are abundant if $B/M \geq [(i+\ell-f)/(\ell-f)]^{1/\gamma}-1$, and in this case the equilibrium is:

$$q_0 = \sigma \left(\frac{\ell - f}{i + \ell - f} \right)^{1/\gamma} \tag{6a}$$

$$q_1 = \sigma \tag{6b}$$

$$p = \frac{1}{1+i} \tag{6c}$$

If, on the other hand, $B/M < [(i+\ell-f)/(\ell-f)]^{1/\gamma} - 1$, then the bond supply is scarce and the equilibrium is:

$$q_0 = \sigma \left(\frac{\ell - f + (1 + B/M)^{-\gamma} f}{i + \ell} \right)^{1/\gamma} \tag{7a}$$

$$q_1 = \sigma \left(\frac{(\ell - f)(1 + B/M)^{\gamma} + f}{i + \ell} \right)^{1/\gamma} \tag{7b}$$

$$p = \frac{1}{1+i} \left(1 - f + f \, \frac{i + \ell}{(\ell - f)(1 + B/M)^{\gamma} + f} \right) \tag{7c}$$

As in Equation (4c), the bond price is not linear in any variable, but depending on the question, it may have other attractive properties. Note that it is independent of the size of the liquidity shock σ, as was the case in Equation (2c) but not (4c). The deep reason is the same as before: if liquidity becomes more valuable then the demand for liquid bonds goes up, but this is exactly offset by the endogenous change in the price level that makes their effective supply go down. Any increase in the value of liquidity falls proportionally on money and bonds.

Finally, we can use Equation (7a) to obtain a formula for the general price level:

$$P = \frac{(i + \ell)^{1/\gamma}}{\sigma} \left[(\ell - f) M^{-\gamma} + f (M + B)^{-\gamma} \right]^{-1/\gamma} \tag{8}$$