

# Monetary Policy, Asset Prices, and Liquidity in Over-the-Counter Markets

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## ABSTRACT

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We develop a dynamic general equilibrium model where agents can allocate their wealth between a liquid asset, which can be used to purchase consumption goods, and an illiquid asset, which represents a better store of value but cannot be used to purchase consumption. Should a consumption opportunity arise, agents may visit a frictional “over-the-counter” secondary asset market where they can exchange illiquid for liquid assets. As a result, such an illiquid asset is imbued with *indirect* liquidity even though it cannot be used directly in exchange for goods. We characterize how monetary policy affects both the issue price and the secondary market price of the asset. We also show that, in contrast to conventional wisdom, search and bargaining frictions in the secondary asset market can improve welfare if inflation is low.

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# 1 Introduction

Since Tobin's (1965) influential work, the effect of monetary policy on asset prices has been a major topic of study in monetary economics. In the majority of papers that have researched this question, the assets under consideration are assumed to trade in competitive markets. However, a recent branch of the finance literature, following the influential work of Duffie, Gârleanu, and Pedersen (2005) (DGP henceforth), has documented that many assets are in fact traded in over-the-counter (OTC) markets, characterized by search and bargaining frictions. The goal of this paper is twofold. First, to revisit the traditional question of the effect of monetary policy on asset prices, within an environment in which assets can be traded in an OTC fashion. Second, to study how the frictions that characterize asset trade can affect social welfare.

To investigate these questions we develop a model economy with an essential role for a liquid asset in the exchange process, in particular fiat money. Agents choose how to allocate their wealth between the liquid asset and a real asset which is illiquid, in the sense that it cannot be used as means of payment. Agents wish to hold liquid portfolios but liquidity comes at a cost, inflation. Alternatively, they can hold the illiquid asset, which typically represents a superior store of value. Should a consumption opportunity arise, the agents may visit a secondary asset market where they can exchange illiquid for liquid assets with agents for whom such an opportunity did not arise. Following the recent literature in finance, we model this market as an OTC market, in which agents meet bilaterally and bargain over the terms of trade. Since the monetary authority controls the cost of holding the liquid asset, it also affects how much agents value visiting the secondary asset market in order to rebalance their liquidity. This is the channel that allows us to link asset pricing in OTC markets to monetary policy.

In our model, the real asset is first traded (issued) in a competitive market, but subsequent trades take place in an OTC fashion.<sup>1</sup> Thus, the *issue price* of the real asset reflects the benefit of the marginal unit held by the representative agent. If the asset supply is sufficiently small, and the cost of holding money is sufficiently large, the issue price carries a *liquidity premium* (i.e. it exceeds the fundamental value), reflecting the asset's property to relax a binding liquidity constraint through trade in the OTC market. As inflation rises, this premium increases because the service that the asset provides, i.e. helping agents *avoid the inflation tax*, becomes more essential. Consistent with empirical evidence, the model predicts that the issue price is higher if the secondary market is more liquid, i.e. if trade in that market is more efficient.<sup>2</sup>

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<sup>1</sup> Assuming that the asset is issued in a competitive market is a methodological device, due to Lagos and Wright (2005), that gives rise to degenerate asset distributions and ensures tractability. However, as we demonstrate in Section 2, many assets that are eventually traded in OTC secondary markets are indeed issued in primary markets with competitive characteristics.

<sup>2</sup> This finding reflects the words of the Assistant Secretary of the Treasury for Financial Markets Brian Roseboro, who states that secondary market liquidity is important because it encourages "more aggressive bidding in the primary market", i.e. a higher issue price ("A Review of Treasury's Debt Management Policy", June 3, 2002, available at <http://www.treas.gov/press/releases/po3149.htm>).

Although the effect of inflation on the primary asset price is straightforward, its effect on the OTC price is ambiguous. Higher inflation generates two opposing forces. On the one hand, agents carry less money, which increases the value of additional liquidity for those who get a consumption opportunity (sellers of assets in the OTC market). These agents are willing to accept a lower price for their assets. On the other hand, the higher inflation tax reduces the value of money, which makes agents without a consumption opportunity (buyers of assets in the OTC market) willing to pay a higher price per unit of asset. Which force dominates depends on bargaining power and on how sensitive real balances are to inflation.<sup>3</sup>

Our model delivers a surprising result regarding welfare. Conventional wisdom suggests that the trading frictions in the OTC market might be associated with welfare losses, and that a competitive market will tend to allocate resources in a more efficient way. In contrast, we find that certain frictions can be welfare enhancing. In particular, we compare our model with a benchmark model where the secondary asset market is competitive. We show that, if inflation is not too high, and if the probability of trading in the secondary market under the two specifications is comparable, then equilibrium welfare is always higher in the model where the secondary asset market is OTC.

The competitive market does not deliver the efficient outcome due to the presence of an externality: when an agent carries an additional dollar she provides insurance against the consumption shock not only to herself, but also to agents who might acquire this dollar in the secondary asset market. However, agents ignore this beneficial effect when choosing their money demand, and this externality is intimately related to the ability of agents to *rebalance* their liquidity in a secondary asset market after the uncertainty regarding consumption has been resolved. When that market is OTC, agents realize that they can only obtain extra liquidity at a cost, which depends on the search and bargaining frictions. Under certain conditions, agents prefer to carry their own money rather than obtain it in the secondary market. Hence, the OTC market frictions can increase the value of the liquid asset which, in turn, allows agents to purchase a higher level of consumption. This is, to the best of our knowledge, the first paper in the OTC asset market literature to show that search and bargaining can actually be *welfare enhancing*.<sup>4</sup>

We also examine the effect of inflation on the volume of OTC trade. We find that this variable is hump-shaped. Intuitively, when inflation is low, agents enter the OTC market with a lot of money, which diminishes their need for additional money, implying that only a small volume of assets will be traded. As inflation increases, agents carry less money, and the volume of trade in the OTC market rises. However, if inflation increases further, the trade volume declines, not

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<sup>3</sup> Empirical evidence seems to verify that the price of common stock (which is typically traded in organized exchanges) is increasing in inflation (see for example Fama (1981)). Our prediction that the OTC asset price could increase or decrease in inflation, depending on bargaining strengths, is a novel theoretical finding that would be interesting to test empirically. However, this is beyond the scope of the present paper.

<sup>4</sup> This result is very topical: the recent U.S. financial reforms (“Dodd-Frank”) include provisions which require trade in certain assets, previously conducted over the counter, to be cleared through exchanges.

because the need of agents to avoid the inflation tax is not strong enough, but because there is not enough liquidity to facilitate asset trade in the OTC market.

This paper extends the DGP framework in two important ways. First, in the DGP framework, gains from trade arise from the assumption that agents have different valuations for the same asset. The authors motivate this assumption by arguing that a seller of the asset has a high need for liquidity. Our model formalizes precisely this idea: all participants in the OTC market have the same valuation for the asset, but the sellers are agents who have a consumption opportunity and, hence, a high need for liquidity. In this way, we provide a micro-founded explanation for a key assumption of the DGP model and, at the same time, an interesting link between financial economics and monetary theory. The second way in which we extend the DGP framework is by removing the “deep pockets” assumption adopted in that paper and in the literature that builds on it. In the DGP model, agents have access to a risk-free real bank account which serves as a liquid asset and ensures that all efficient trades will be consummated. In our model, liquidity comes at a cost, inflation, and, thus, monetary policy can affect OTC asset trade in non-trivial ways. For instance, in a regime of high inflation, trade may be rationed due to a lack of liquidity, a result that could not be obtained in the DGP framework.

Our paper provides a framework that integrates the different definitions of asset liquidity in monetary economics and finance. As Lagos (2008) points out, “there seems to be [...] no definition of ‘liquidity’ that is generally agreed upon”. In monetary theory, liquidity is an attribute of an asset, and it refers to how easily the asset can be used to finance consumption opportunities. In finance, on the other hand, liquidity is an attribute of a market, and it refers to how easily an investor can find a counterparty for trade, and at what cost.<sup>5</sup> Our model bridges this gap: the frictions associated with search and bargaining in the OTC market determine how easily the asset can be transformed into the medium of exchange and, hence, into consumption.

## 1.1 Related Literature

Several papers in recent literature also introduce a real asset in monetary models and find that the asset price can include a liquidity premium, which is increasing in inflation. Examples include Geromichalos, Licari, and Suarez-Lledo (2007), Lagos (2011), Lester, Postlewaite, and Wright (2012), Nosal and Rocheteau (2012), Jacquet and Tan (2012), and Rocheteau and Wright (2012).<sup>6</sup> An important difference is that in these papers the asset carries a liquidity premium because it competes *directly* with money as a medium of exchange, while in our model, the asset carries a premium because it can help agents increase their liquidity *indirectly*, when swapped

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<sup>5</sup> A representative example from the monetary-search literature is Lagos and Wright (2005). For the finance-related definition, see DGP or Lagos and Rocheteau (2009).

<sup>6</sup> There is also a number of papers which study the liquidity properties of assets other than fiat money without focusing on the effect of inflation on asset prices. Examples include Lagos and Rocheteau (2008), Li, Rocheteau, and Weill (2011), Andolfatto and Martin (2012), and Andolfatto, Berentsen, and Waller (2013).

with money in the OTC market. Another difference is that in our model the liquidity premium vanishes for high levels of inflation: when inflation is high, agents carry so little money that even a small supply of the asset can be enough to acquire all the available real balances, and the marginal unit of the asset is only useful as a store of value, which is to say that the asset is priced at fundamental. Hu and Rocheteau (2012) arrive at a similar result in a model where assets compete directly with money as media of exchange, and the trading mechanism in bilateral meetings in the decentralized market is designed to maximize social welfare subject to certain frictions of the environment.

Some recent papers exploit the idea that assets prices can carry liquidity premia and offer a new perspective for looking at long-standing asset-related puzzles, see for example Lagos (2010) (equity premium puzzle) and Geromichalos and Simonovska (2014) (asset home bias puzzle). In these papers, assets serve as media of exchange, an assumption which might be subject to criticism as, typically, we do not observe agents trade assets directly for goods. Lagos (2011) argues that assets can carry liquidity premia in broader contexts: for example, they may serve as collateral or help agents enter a repurchase agreement.<sup>7</sup> We view our analysis as taking this argument a step further. The key liquidity property of an asset in our model is that it can be sold before maturity, in exchange for money, and can therefore facilitate trade without itself serving as a medium of exchange, as collateral, or in a repurchase agreement.

Our paper is also related to a number of papers that explore the idea that agents can rebalance their money holdings after a consumption opportunity arises. Examples of such papers include Kocherlakota (2003), Boel and Camera (2006), Berentsen, Camera, and Waller (2007), and Berentsen and Waller (2011). Within this class of papers, the one that is closer to our work is Li and Li (2013), who introduce a real asset which serves as collateral and consider an environment where competitive banks channel funds from people with idle cash to those who need liquidity to finance unanticipated consumption. The authors show that, if exclusion from the credit system is feasible, inflation relaxes agents' credit constraints (by increasing the cost of default), so that an increase in inflation can potentially increase loan-to-value ratios, liquidity, and, consequently, the output of the economy.

There is a number of recent papers that incorporate OTC asset trade within a monetary model. Lagos and Zhang (2013) study a model where agents have a different, exogenously given, valuation for the same asset (as in DGP), and money serves as a medium of exchange in a market where low-valuation agents wish to sell their assets to high-valuation agents. In our framework, all agents have the same asset valuation, but sellers of the asset are agents who have a greater need for liquidity. Hence, the sources that generate gains from trade in the two models are very different. Nosal and Mattesini (2013) consider an environment where agents visit

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<sup>7</sup>For example, Venkateswaran and Wright (2013) and Geromichalos, Lee, Lee, and Oikawa (2014) study the liquidity properties of assets which can be used as collateral to secure loans, within environments in which borrowers' commitment to repay their debt is limited.

an OTC market in order to rebalance their liquidity (like in our paper), but they focus on the role of dealers. Trejos and Wright (2012) develop a framework that nests the DGP model into a “second-generation” monetary-search model (e.g. Shi (1995) and Trejos and Wright (1995)) and discuss similarities and differences between the two literatures. Geromichalos, Herrenbrueck, and Salyer (2013) extend the present model by incorporating assets with different maturities and use their framework to rationalize an upward sloping yield curve. Finally, Herrenbrueck (2014) studies quantitative easing in a continuous-time framework which embeds the DGP model in the monetary economy of Rocheteau, Weill, and Wong (2014). Because asset liquidation is subject to frictions, money and financial assets are imperfect substitutes, which gives scope to policies that alter the supply of these assets in order to affect the real economy.

As we have already mentioned, our paper offers a micro-founded explanation for the assumption that agents have different valuations for the same asset, which is adopted by DGP and the majority of papers that build on their framework (see, for example, Vayanos and Weill (2006), Lagos, Rocheteau, and Weill (2011), and Chiu and Koepl (2011)). Duffie, Gârleanu, and Pedersen (2007) and Gârleanu (2009) offer an alternative micro-founded interpretation of the assumption in question. In these papers, investors are risk averse, and asset returns are correlated with their endowments. Then, a low valuation type is just an investor whose asset is positively correlated with the endowment process and, hence, a poor hedging device.<sup>8</sup>

The remainder of the paper is organized as follows. Section 2 describes the physical environment. Section 3 analyzes the terms of trade in the OTC market and the optimal behavior of agents. Section 4 describes a steady state equilibrium and examines the effects of monetary policy on asset prices and welfare. Section 5 concludes.

## 2 Physical Environment

Our model extends the framework of Lagos and Wright (2005). Time is discrete with an infinite horizon, and each period consists of three sub-periods. During the first sub-period, economic activity takes place in a traditional Walrasian or centralized market. We refer to this market as the CM. In the second sub-period, a secondary asset market opens, which resembles the Over-the-Counter markets of DGP. We refer to this market as the OTC market. In the third sub-period agents visit a decentralized competitive market for goods, similar to Rocheteau and Wright (2005). We refer to this market as the DM. A detailed description of these markets will follow. There are two types of agents, buyers and sellers, depending on their role in the DM. All agents live forever and their types are permanent. The measure of buyers is normalized to

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<sup>8</sup> DGP offer several interpretations of the investor types. In their own words, “[...] a low-type investor may have (i) low liquidity (that is, a need for cash), (ii) high financing costs, (iii) hedging reasons to sell, (iv) a relative tax disadvantage, or (v) a lower personal use of the asset”. Hence, the aforementioned papers provide a micro-founded interpretation based on (iii), while our paper explores the avenue suggested in (i).

the unit. The measure of sellers will not be crucial for any of the results (there is no free entry), and it is discussed below.

All agents discount the future between periods (but not sub-periods) at rate  $\beta \in (0, 1)$ . Buyers consume in the first and the third sub-periods and supply labor in the first sub-period. Their preferences for consumption and labor within a period are given by  $\mathcal{U}(X, H, q)$ , where  $X, H$  represent consumption and labor in the CM, respectively, and  $q$  consumption in the DM. Sellers consume only in the CM, and they produce in both the CM and the DM. Their preferences are given by  $\mathcal{V}(X, H, h)$ , where  $X, H$  are as above, and  $h$  stands for hours worked in the DM. Following Lagos and Wright (2005), we adopt the functional forms

$$\begin{aligned}\mathcal{U}(X, H, q) &= X - H + u(q), \\ \mathcal{V}(X, H, h) &= X - H - c(h).\end{aligned}$$

We assume that  $u$  is twice continuously differentiable with  $u(0) = 0$ ,  $u' > 0$ ,  $u'(0) = \infty$ ,  $u'(\infty) = 0$ , and  $u'' < 0$ . For simplicity, we set  $c(h) = h$ , but this is not crucial for any results. Let  $q^*$  denote the first-best level of production in the DM, i.e.,  $q^* \equiv \{q : u'(q^*) = 1\}$ . Notice that there is no consumption or production in the OTC market. The role of this market is to allow agents to re-balance their portfolios before entering the DM.

In the first sub-period, all agents consume and produce a general good or fruit. Agents have access to a technology that transforms one unit of labor into one unit of the fruit. In every period a new set of trees are born that produce fruit, as in Lucas (1978). Agents can purchase shares of these trees at the ongoing market price  $\psi_t$ . The supply of these trees is denoted by  $A > 0$ , and it is fixed over time. Each unit of the tree delivers  $d$  units of fruit in the next period, and then it dies. Hence, the maturity of the real asset is one period. The second asset that is traded in the CM is fiat money. The market price of money is denoted by  $\varphi_t$ . Its supply is controlled by a monetary authority, and it evolves according to  $M_{t+1} = (1 + \mu)M_t$ , with  $\mu > \beta - 1$ . New money is introduced, or withdrawn if  $\mu < 0$ , via lump-sum transfers to buyers in the CM. Money has no intrinsic value, but it is portable, storable, divisible, and recognizable by all agents. Hence, it can serve as a medium of exchange in the DM, and help bypass the frictions created by anonymity and the lack of a double coincidence of wants.

After leaving the CM, a measure  $\ell < 1$  of buyers finds out that they will be able to consume in the forthcoming DM. We refer to these buyers as the C-types, and to the remaining  $1 - \ell$  buyers as the N-types. As we discuss below, the only asset that can serve as means of payment in the DM is fiat money. Since C-type and N-type buyers are ex-ante identical, the latter hold some cash that they will not use in the current period. This gives rise to potential gains from trade which can be exploited in the OTC market, as the C-types can readjust their liquidity by selling assets for money, and the N-types are the providers of liquidity. We denote the total measure of matches by  $f \leq \min\{\ell, 1 - \ell\}$ , and we maintain the assumption  $f > 0$ . (This measure  $f$  could represent a matching function  $f(\ell, 1 - \ell)$  which brings together C-types and N-types.)

Within each match, the terms of trade are determined through proportional bargaining, following Kalai (1977), and we let  $\lambda \in [0, 1]$  represent the C-type’s bargaining power.

The third sub-period is a competitive decentralized market as in Rocheteau and Wright (2005). C-type buyers meet with sellers, both take prices as given, and supply and demand determine a competitive price. Due to anonymity and a lack of record keeping, settlement has to be immediate, and we assume that sellers in this market cannot verify the authenticity of assets other than fiat money, which is therefore the only feasible means of payment (examples of papers which relax this assumption and allow both money and real assets to be used as media of exchange include Geromichalos *et al* (2007) and Lagos and Rocheteau (2008)). This assumption is consistent with one of the goals of the paper, which is to study whether assets can exhibit *indirect* liquidity properties *even when they do not serve as media of exchange*.<sup>9</sup> The assumption that the DM is a perfectly competitive market is made in the interest of keeping the setup of this market as simple as possible, and focusing on the search and bargaining frictions in the OTC market, which is the central question of our paper.<sup>10</sup>

The framework presented here allows us to study asset pricing, and its link to monetary policy, both within traditional, i.e. Walrasian, markets and within OTC markets *à la* DGP. Assuming that the asset is issued in a Walrasian market is a methodological innovation, due to Lagos and Wright (2005), which (together with quasi linearity) gives rise to degenerate asset holding distributions and, hence, ensures tractability. However, many assets that are eventually traded in OTC secondary markets are indeed issued in primary markets with more competitive characteristics. For instance, US Treasury Bills are issued through single-price auctions, in order “to minimize the government’s costs [...] by promoting broad, competitive bidding” (Garbade and Ingber (2005)).

### 3 Value Functions and Optimal Behavior

#### 3.1 Value Functions

We begin with the description of the value functions in the *CM*. Consider first a buyer who enters this market with money and asset holdings  $(m, a)$ . The Bellman equation is given by

$$W^B(m, a) = \max_{X, H, \hat{m}, \hat{a}} \{X - H + \mathbb{E}_i \{\Omega^i(\hat{m}, \hat{a})\}\}$$

$$\text{s.t. } X + \varphi \hat{m} + \psi \hat{a} = H + \varphi(m + \mu M) + da,$$

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<sup>9</sup> There might be deeper reasons why agents prefer to use money versus financial assets in order to carry out transactions. For example, Rocheteau (2011) and Lester, Postlewaite, and Wright (2012) consider environments that do not place any restrictions on what objects agents can use as media of exchange. Both papers show that, under asymmetric information, fiat money can endogenously arise as a superior medium of exchange.

<sup>10</sup> Alternatively, one could assume that trade in the DM is subject to search-and-bargaining frictions as in Lagos and Wright (2005), but that all C-type buyers match with a seller and that the buyers make a take-it-or-leave-it offer. This would not affect any of our results.

where variables with hats denote next period's choices, and  $\mathbb{E}$  denotes the expectations operator. All prices are expressed in terms of the general good. The function  $\Omega^i$  represents the value function in the OTC market for a buyer of type  $i = \{C, N\}$ , to be described in more detail below. At any the optimum,  $X$  and  $H$  are indeterminate but their difference is not. Using this fact and substituting  $X - H$  from the budget constraint into  $W^B$  yields

$$W^B(m, a) = \varphi(m + \mu M) + da + \max_{\hat{m}, \hat{a}} \{-\varphi\hat{m} - \psi\hat{a} + \mathbb{E}_i \{\Omega^i(\hat{m}, \hat{a})\}\}. \quad (1)$$

As is standard in models that build on Lagos and Wright (2005), the optimal choice of the agent does not depend on the current state (due to the quasi-linearity of  $\mathcal{U}$ ), and the CM value function is linear. We collect all the terms in (1) that do not depend on the state variables  $m, a$  and write

$$W^B(m, a) = \varphi m + da + \Upsilon^B, \quad (2)$$

where the definition of  $\Upsilon^B$  is obvious.

Next, consider a seller's value function in the CM. Since in this model we assume that  $\mu > \beta - 1$ , the cost of carrying money (the nominal interest rate) is strictly positive, and the sellers will choose not to carry any money as they leave the CM. A similar argument can be used to claim that sellers will also not hold the real asset as they leave the CM.<sup>11</sup> Hence, when a seller enters the CM, she can only hold money that she received during trade in last period's DM, and the CM value function is given by

$$\begin{aligned} W^S(m) &= \max_{X, H} \{X - H + V^S\} \\ \text{s.t. } &X = H + \varphi m, \end{aligned}$$

where  $V^S$  denotes the seller's value function in the forthcoming DM. We can again use the budget constraint to substitute  $X - H$  and show that  $W^S$  will be linear:

$$W^S(m) = \varphi m + V^S \equiv \Upsilon^S + \varphi m. \quad (3)$$

After leaving the CM, and before the OTC market opens, buyers learn whether they will have a chance to access this period's DM (C-types) or not (N-types). This chance will occur with probability  $\ell \in (0, 1)$ , and by the law of large numbers, a proportion  $\ell$  of buyers will be of type C. Thus, the expected value for a buyer with portfolio  $(m, a)$ , before she enters the OTC

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<sup>11</sup> A careful proof of the result that sellers do not hold any money can be found in Rocheteau and Wright (2005). Regarding the optimal choice of real assets, as we shall see in what follows, there exist equilibria in which the cost of holding the real asset is zero. In this case, the seller is indifferent between holding zero or some positive amount of the asset. However, if we assume that there is a fixed cost,  $c > 0$ , of participating in the Walrasian assets' market, then holding zero assets is the unique optimal choice for the seller, even for an infinitesimally small  $c$ .

market is given by

$$\mathbb{E}_i \{ \Omega^i(m, a) \} = \ell \Omega^C(m, a) + (1 - \ell) \Omega^N(m, a). \quad (4)$$

In the OTC market, C-type buyers (who may want additional money) are matched with N-type buyers (who want to mitigate the inflation tax on their money holdings) at total matching rate  $f$ . Given that matching is random, the matching probabilities (or arrival rates) are  $f/\ell$  (for C-types) and  $f/(1-\ell)$  (for N-types). Let  $\chi$  denote the units of asset that the C-type transfers to the N-type, and  $\psi_I$  the monetary price, per unit of asset, at which this transfer takes place (so that  $\chi\psi_I$  stands for the total units of money received by the C-type). These terms will be determined through bargaining in Section 3.3. We have

$$\Omega^C(m, a) = \frac{f}{\ell} V^B(m + \chi\psi_I, a - \chi) + \left(1 - \frac{f}{\ell}\right) V^B(m, a), \quad (5)$$

$$\Omega^N(m, a) = \frac{f}{1-\ell} \beta W^B(m - \chi\psi_I, a + \chi) + \left(1 - \frac{f}{1-\ell}\right) \beta W^B(m, a), \quad (6)$$

where  $V^B$  denotes a buyer's value function in the DM. Notice that N-type buyers proceed directly to next period's CM.

Lastly, consider the value functions in the DM. Let  $q$  denote the quantity of goods traded, and  $p$  their price in units of fiat money. These terms will be determined in Section 3.2. The DM value function for a buyer who enters that market with portfolio  $(m, a)$  is given by

$$V^B(m, a) = u(q) + \beta W^B(m - pq, a), \quad (7)$$

and the DM value function for a seller (who enters with no money or assets) is given by

$$V^S = -q + \beta W^S(pq).$$

Having established the agents' value functions, we now proceed to the description of the terms of trade in the DM and the OTC market.

### 3.2 Price determination in the DM

We proceed by backward induction and begin with the decentralized competitive market in the last subperiod. Consider first the problem of a C-type buyer with money holdings  $m$  and real asset holdings  $a$ , taking the nominal price of goods,  $p$ , as given:

$$\begin{aligned} \max_q \{ & u(q) + \beta W^B(m - pq, a) \} \\ \text{s.t. } & 0 \leq q \leq m/p. \end{aligned}$$

Substituting the value function  $W^B$  from (2) allows us to re-write the buyer's problem as

$$\begin{aligned} \max_q \{ & u(q) - \beta \hat{\varphi} p q + \beta W^B(m, a) \} \\ \text{s.t. } & 0 \leq q \leq m/p. \end{aligned}$$

Notice that the relevant value of money is  $\hat{\varphi}$ , discounted by  $\beta$ , because the forthcoming CM opens in the next period.

Consider next the problem of a seller who, in the beginning of the DM sub-period, holds no money or assets. The seller decides on the quantity of hours  $h$  to work, knowing that one hour of work produces one unit of goods  $q$ , taking the price  $p$  as given:

$$\begin{aligned} \max_{h,q} \{ & -h + \beta W^S(pq) \} \\ \text{s.t. } & 0 \leq q \leq h. \end{aligned}$$

Substituting the value function  $W^S$  from (3), and  $h = q$ , we can write:

$$\begin{aligned} \max_{p,q} \{ & -q + \beta \hat{\varphi} p q + \beta W^S(0) \} \\ \text{s.t. } & q \geq 0. \end{aligned}$$

As the measure of both sellers and C-type buyers is  $\ell$ , market clearing requires that the buyer's and seller's problems yield the same  $q$ .

**Lemma 1.** *Define the amount of money that, given tomorrow's value of money  $\hat{\varphi}$ , allows the buyer to purchase the first-best  $q^*$ , i.e.  $m^* = q^*/(\beta \hat{\varphi})$ . Then, the competitive solution to the buyer's and seller's problems is given by  $p(m) = 1/(\beta \hat{\varphi})$  and  $q(m) = \beta \hat{\varphi} \min\{m, m^*\}$ .*

*Proof.* See the appendix. □

As the seller's cost function is linear, the competitive problem is straightforward. The only variable that affects the solution is the representative buyer's money holdings. As long as the buyer carries  $m^*$  or more, the first-best quantity  $q^*$  will be exchanged. On the other hand, if  $m < m^*$ , then buyers do not have enough cash to induce the sellers to produce  $q^*$ . The cash constrained buyer will spend all her money and the seller will produce the quantity of good that satisfies her participation constraint.

### 3.3 Bargaining in the OTC market

We now study the terms of trade in the OTC market. Although buyers are ex ante identical, only C-types will get an opportunity to consume in the forthcoming DM, and may be willing

to trade assets for money. The N-type buyers cannot access the DM in this period and have no use for money (other than as a store of value), so they may be willing to trade money for assets. Hence, although the OTC is an asset market, all trades are driven by agents' different valuations for money. In this way, we formalize DGP's suggestion that asset trade is generated by the varying needs for liquidity of market participants, and we provide micro-foundations for their assumption that agents have different valuations for the same asset.

Consider a meeting in the OTC market between a C-type with money and asset holdings  $(m, a)$ , respectively, and an N-type with portfolio  $(\tilde{m}, \tilde{a})$ . Let  $\chi$  represent the amount of assets that are transferred from the C-type to the N-type (in principle, this variable may be negative), and let  $\psi_I$  denote the monetary price per unit of asset that is paid to the seller of assets. We will assume that the two parties split any potential surplus according to Kalai's proportional (or egalitarian) bargaining solution.<sup>12</sup> Letting  $\lambda \in [0, 1]$  denote the C-type's bargaining power, we can summarize the bargaining solution as follows:

**Lemma 2.** *Define the cutoff level of asset holdings*

$$\bar{a}(m, \tilde{m}) \equiv \begin{cases} \frac{1}{\beta d} \{ (1 - \lambda) \{ u[\beta \hat{\varphi}(m + \tilde{m})] - u(\beta \hat{\varphi}m) \} + \lambda \beta \hat{\varphi} \tilde{m} \}, & \text{if } m + \tilde{m} < m^*, \\ \frac{1}{\beta d} \{ (1 - \lambda) [u(\beta \hat{\varphi}m^*) - u(\beta \hat{\varphi}m)] + \lambda \beta \hat{\varphi}(m^* - m) \}, & \text{if } m + \tilde{m} \geq m^*. \end{cases}$$

Then the solution to the bargaining problem is given by

$$\chi(m, \tilde{m}, a) = \begin{cases} \bar{a}(m, \tilde{m}), & \text{if } a \geq \bar{a}(m, \tilde{m}), \\ a, & \text{if } a < \bar{a}(m, \tilde{m}). \end{cases} \quad (8)$$

$$\psi_I(m, \tilde{m}, a) = \begin{cases} \frac{\min\{m^* - m, \tilde{m}\}}{\bar{a}(m, \tilde{m})}, & \text{if } a \geq \bar{a}(m, \tilde{m}), \\ \psi_I^a(m, a), & \text{if } a < \bar{a}(m, \tilde{m}), \end{cases} \quad (9)$$

where  $\psi_I^a(m, a)$  solves

$$(1 - \lambda) \{ u[\beta \hat{\varphi}(m + a\psi_I^a)] - u(\beta \hat{\varphi}m) \} + \lambda \beta \hat{\varphi} a \psi_I^a = \beta d a. \quad (10)$$

*Proof.* See the appendix. □

Lemma 2 has a very intuitive interpretation. If  $\tilde{m} \geq m^* - m$ , the C-type should receive  $m^* - m$ , which will allow her to purchase  $q^*$  in the forthcoming DM. On the other hand, if  $\tilde{m} < m^* - m$ , the constrained optimal requires the N-type to hand over all her money to the

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<sup>12</sup> Proportional bargaining differs from Nash bargaining (Nash Jr (1950)) in that it relaxes the axiom of scale invariance and replaces it with an axiom of monotonicity: "if additional options were made available to the individuals in a given situation then no one of them should lose utility because of the availability of these new options" (Kalai (1977)). We prefer the Kalai bargaining solution because it makes our model much more tractable by eliminating hold-up problems. For a comprehensive discussion, see Aruoba, Rocheteau, and Waller (2007).

C-type. However, one needs to worry about whether the C-type has enough assets to compensate the N-type for these transfers of liquidity. This critical level of assets depends on whether  $m + \tilde{m}$  exceeds  $m^*$  or not, and it is given by the term  $\bar{a}(m, \tilde{m})$ . If  $a \geq \bar{a}(m, \tilde{m})$ , the asset constraint does not bind. In this case, the total money transfer is  $\chi\psi_I = \min\{m^* - m, \tilde{m}\}$ , and the C-type gives up exactly  $\bar{a}(m, \tilde{m})$  units of the asset. On the other hand, if  $a < \bar{a}(m, \tilde{m})$ , the C-type gives away all her assets,  $\chi = a$ , and the per unit price is determined such that the sharing rule of the surplus between the two parties (equation (10)) is satisfied. Notice that the bargaining solution is never affected by the N-type's asset holdings.<sup>13</sup>

In the appendix, we prove some interesting properties of the bargaining solution. For instance, it is shown that  $\partial\chi/\partial\lambda > 0$ . In words, as the C-type's bargaining power increases, the amount of assets that she has to give up in order to get the first-best amount of money,  $m^* - m$ , goes down. Moreover, we show that  $\partial\psi_I/\partial\lambda > 0$ , i.e. when the bargaining power of the C-type goes up, the price that she obtains for every unit of asset sold also increases. Finally, it is shown that  $\partial\psi_I/\partial m > 0$ , which is also a fairly intuitive result: Since  $\psi_I$  denotes the price at which the C-type sells her assets, the more money she carries, the closer she is to  $m^*$ , and the less willing she will be to accept a low price. On the contrary, if  $m$  is very small, the C-type is more desperate for a monetary transfer by the N-type and more likely to accept a low price for her assets.

Having solved for the trading outcomes in the OTC and DM, we now proceed to derive the objective function of the buyer and describe optimal behavior.

### 3.4 Objective Function and Optimal Behavior

The goal of this subsection is to describe the buyer's optimal choice of  $(\hat{m}, \hat{a})$ . To that end, substitute (5) and (6) into (4), and lead the emerging expression for  $\mathbb{E}_i\Omega^i$  by one period to obtain

$$\begin{aligned} \mathbb{E}_i \{ \Omega^i(\hat{m}, \hat{a}) \} &= f V^B(\hat{m} + \chi\psi_I, \hat{a} - \chi) + [\ell - f] V^B(\hat{m}, \hat{a}) \\ &\quad + f\beta W^B(\hat{m} - \tilde{\chi}\tilde{\psi}_I, \hat{a} + \tilde{\chi}) + [1 - \ell - f]\beta W^B(\hat{m}, \hat{a}). \end{aligned} \quad (11)$$

The four terms in this expression represent the benefit for a buyer who holds a portfolio of  $(\hat{m}, \hat{a})$  and turns out to be a matched C-type, an unmatched C-type, a matched N-type, or an unmatched N-type, respectively. It is understood that the expressions  $\chi, \psi_I, \tilde{\chi}$ , and  $\tilde{\psi}_I$  are de-

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<sup>13</sup> It should be noted that the suggested solution assumes that trade in the OTC takes place only if a strictly positive surplus is generated. For example, consider the case in which  $m + \tilde{m} \geq m^*$ , and assume that  $a$  is unlimited. Strictly speaking, any  $\chi\psi_I \geq m^* - m$  is part of the bargaining solution. However, when an amount of money  $\chi\psi_I > m^* - m$  changes hands, the C-type and the N-type are just swapping assets for money, and no surplus is generated, since the C-type can already purchase  $q^*$  in the DM. To keep the analysis simple in the forthcoming sections, we assume that in cases like this  $\chi\psi_I = m^* - m$ .

scribed by the solution to the OTC bargaining problem (Lemma 2), and in particular

$$\begin{aligned}\chi &= \chi(\hat{m}, \tilde{m}, \hat{a}), & \psi_I &= \psi_I(\hat{m}, \tilde{m}, \hat{a}), \\ \tilde{\chi} &= \chi(\tilde{m}, \hat{m}, \tilde{a}), & \tilde{\psi}_I &= \psi_I(\tilde{m}, \hat{m}, \tilde{a}).\end{aligned}$$

In these expressions, the first argument represents the C-type's money holdings, the second argument represents the N-type's money holdings, and the third argument stands for the C-type's asset holdings. Terms with tildes represent the buyer's beliefs about her potential counterparty's money and asset holdings in the OTC market.

The next step is to substitute the value functions  $W^B$  and  $V^B$  from (2) and (7), respectively, into (11). Insert the emerging expression into (1), and focus on the terms inside the maximum operator of (1) (i.e. ignore the terms that do not affect the choice variables). We will define this expression as  $J(\hat{m}, \hat{a})$ , and we will refer to it as the buyer's *objective function*. After some manipulations, one can verify that<sup>14</sup>

$$\begin{aligned}J(\hat{m}, \hat{a}) &= -\varphi\hat{m} - \psi\hat{a} \\ &+ f \{u[\beta\hat{\varphi}(\hat{m} + \chi\psi_I)] + \beta d(\hat{a} - \chi)\} + (\ell - f) [u(\beta\hat{\varphi}\hat{m}) + \beta d\hat{a}] \\ &+ f \left[ \beta\hat{\varphi}(\hat{m} - \tilde{\chi}\tilde{\psi}_I) + \beta d(\hat{a} + \tilde{\chi}) \right] + (1 - \ell - f) [\beta\hat{\varphi}\hat{m} + \beta d\hat{a}],\end{aligned}\tag{12}$$

where the first line represents the cost of purchasing  $\hat{m}$  units of money and  $\hat{a}$  units of the real asset, and the last four terms admit a similar interpretation as equation (11).

We relegate technical details of the representative buyer's optimal choice of  $(\hat{m}, \hat{a})$  to Appendix A.1 and turn to general equilibrium.

## 4 Equilibrium

In this section we describe equilibrium, with a special focus on the effects of monetary policy on equilibrium asset prices and welfare. We restrict attention to symmetric, steady state equilibria, where all agents choose the same portfolios, and the real variables of the model remain constant over time.<sup>15</sup> Since, in steady state, real money balances do not change over time, we have  $\varphi M = \hat{\varphi} \hat{M}$ , implying that  $\varphi/\hat{\varphi} = 1 + \mu$ . Before stating the definition of a steady state equilibrium,

<sup>14</sup>To arrive at (12), we have used the fact that, in the DM, we will always be on the constrained branch of the competitive solution, i.e.  $q(m) = \beta\hat{\varphi}m$ , regardless of whether the C-type got matched in the OTC or not. An unmatched C-type never enters the DM with money holdings greater than  $m^*$ , because the nominal interest rate is always positive. On the other hand, a matched C-type will never hold money in excess of  $m^*$ , because we have ruled out trades in the OTC that do not generate a positive surplus (see proof of Lemma 2 or footnote 13).

<sup>15</sup>As we establish in Appendix A.1, the optimal choice of money holdings is always unique, hence, all agents will choose the same  $\hat{m}$ . On the other hand, the choice of  $\hat{a}$  might not be unique (when  $\psi = \beta d$ ). In this case the symmetry assumption is non-trivial.

it is important to notice that symmetry rules out regions of  $(\hat{m}, \hat{a})$ -space in which C-types and N-types hold different portfolios, since a C-type and an N-type buyer are ex ante identical. In our description of individually optimal behavior (Appendix A.1 and Figure 8), we labeled the regions in order from 1-5; under symmetry, Regions 1, 3, and 5 remain, whereas Regions 2 and 4 are ruled out.

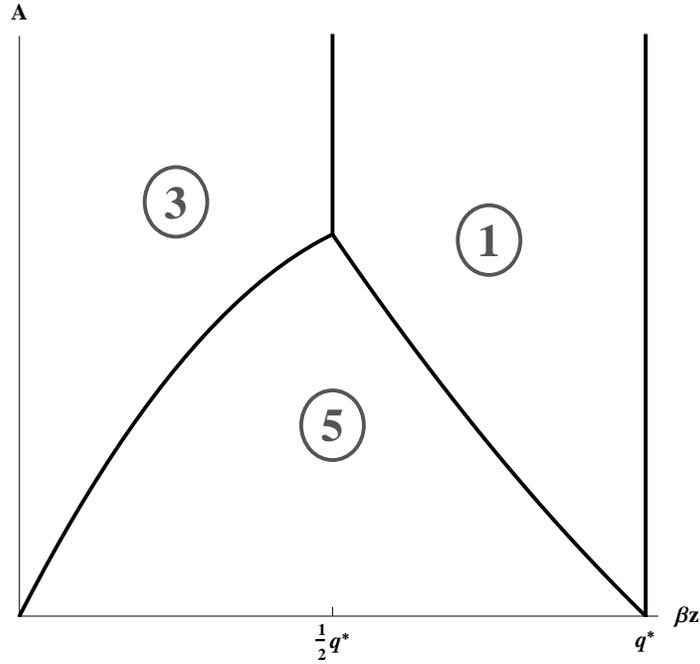


Figure 1: Aggregate regions of equilibrium, in terms of real balances.

1. Agents carry enough money and assets so that, when matched in the OTC market, the C-type can acquire sufficient liquidity in order to achieve the first-best in the DM.
3. Agents carry so little money relative to assets that, when matched in the OTC, the N-type will sell all of her money but the C-type will not achieve the first-best in the DM.
5. Agents carry so few assets that, when matched in the OTC, the C-type will sell all of her assets but not obtain enough money in order to achieve the first-best in the DM.

These regions are described in Figure 1, and we will refer to them as the “aggregate regions”, as opposed to the “individual regions” in Figure 8 in the appendix.

**Definition 1.** A steady state equilibrium is a list  $\{\psi, z, q_1, q_2, \chi, \psi_I\}$ , where  $z = \varphi M$  represents the real money balances,  $q_2$  stands for the amount of good exchanged in the DM, when the buyer was matched in the preceding OTC market, and  $q_1$  is the analogous expression for the case of a buyer who was not matched in the OTC. The equilibrium objects are such that:

- i. The representative buyer behaves optimally under the equilibrium prices  $\psi$  and  $\varphi$ . Moreover,  $\varphi/\hat{\varphi} = 1 + \mu$ .
- ii. The equilibrium quantity  $q_1$  satisfies  $q_1 = \beta\varphi M = \beta z$ , and the quantity  $q_2$  is defined as the following function of  $q_1$ :

$$q_2(q_1) = \begin{cases} q^*, & \text{in Region 1,} \\ 2q_1, & \text{in Region 3,} \\ \tilde{q}(q_1), & \text{in Region 5,} \end{cases}$$

where  $\tilde{q}$  solves  $(1 - \lambda)[u(\tilde{q}) - u(q_1)] + \lambda(\tilde{q} - q_1) = \beta dA$ .

- iii. The terms of OTC trade  $(\chi, \psi_I)$  satisfy (8) and (9).
- iv. Markets clear and expectations are rational:  $\hat{m} = \tilde{m} = (1 + \mu)M$ , and  $\hat{a} = \tilde{a} = A$ .

**Lemma 3.** *A steady state equilibrium  $\{\psi, z, q_1, q_2, \chi, \psi_I\}$  exists and  $\{z, q_1, q_2, \chi, \psi_I\}$  are unique.*

*Proof.* See the appendix. □

The definition of equilibrium is straightforward. Notice that the equilibrium quantity of goods produced in the DM depends on whether the buyer was matched in the preceding OTC market or not, since this critically affects her real balances. A buyer who did not match in the OTC, will be able to purchase  $q_1 < q^*$  from the seller. How much good the matched (in the OTC) buyer can purchase, depends on whether  $q_1$  is greater than or smaller than  $q^*/2$ , and whether the representative buyer has enough assets to purchase the optimal amount of real balances. Of course, in equilibrium, the latter only depends on the exogenous supply,  $A$ .

Lemma 3 guarantees existence of equilibrium, and states that the equilibrium asset price might not be unique for certain parameter values. We will discuss this issue in detail below. Ultimately, we wish to examine how the various equilibrium objects depend on the asset supply,  $A$ , and the policy parameter,  $\mu$ . This task is performed in Propositions 1, 2, and 3 below. As an intermediate step, it is useful to describe precisely which aggregate region (in terms of real balances and asset supply, as shown in Figure 1) equilibrium will lie in, for different values of  $A$  and  $\mu$ . The following lemma provides the details.

**Lemma 4.** *Define the level of aggregate supply  $\bar{A} \equiv \{(1 - \lambda)[u(q^*) - u(q^*/2)] + \lambda q^*/2\}(\beta d)^{-1}$ .*

**Case 1:** *If  $A \geq \bar{A}$ , then there exists a cutoff  $\tilde{\mu} > \beta - 1$  such that:*

- i. *If  $\mu \in (\beta - 1, \tilde{\mu})$ , equilibrium is in the interior of Region 1;*
- ii. *if  $\mu \in (\tilde{\mu}, \infty)$ , equilibrium is in the interior of Region 3.*

**Case 2:** If  $A < \bar{A}$ , then there exist cutoffs  $\mu', \mu'', \mu'''$ , with  $\beta - 1 < \mu' < \mu'' < \mu'''$ , such that:

- i. if  $\mu \in (\beta - 1, \mu')$ , equilibrium is in the interior of Region 1;
- ii. if  $\mu \in (\mu', \mu'')$ , equilibrium is in the interior of Region 5;
- iii. if  $\mu \in [\mu'', \mu''']$ , equilibrium is on the boundary of Regions 5 and 3;
- iv. if  $\mu \in (\mu''', \infty)$ , equilibrium is in the interior of Region 3.

*Proof.* See the appendix. □

Lemma 4 can be explained easily with the assistance of Figure 2. From the analysis of the

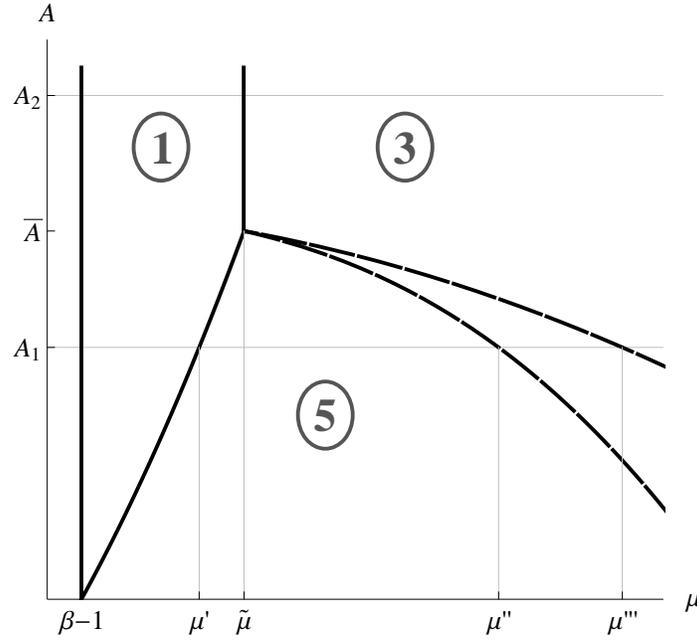


Figure 2: Aggregate regions of equilibrium, in terms of money growth.

buyer's optimal behavior, we know that the demand for money declines as a function of inflation.<sup>16</sup> If the asset supply is relatively high, more precisely if  $A \geq \bar{A}$ , rising inflation pushes agents first into Region 1 and then into Region 3. On the other hand, if the asset supply is low, i.e.  $A < \bar{A}$ , rising inflation pushes agents first into Region 1, then into Region 5 (where the asset is scarce), and finally into Region 3 (where the asset is relatively abundant). It is important to observe that, for every  $A < \bar{A}$ , the region  $[\mu'', \mu''']$  represents the *boundary* between Regions 3 and 5 in terms of Figure 1. The reason why this boundary has a positive measure in Figure 2 is a kink in the objective function on this boundary, resulting in a discontinuity of the money and asset demand functions. We discuss this in detail in Appendix A.1.3.

<sup>16</sup> In Figure 9 in the appendix, we plot money demand against  $\varphi/(\beta\hat{\varphi})$ , which in steady state equals  $(1 + \mu)/\beta$ .

## 4.1 Characterization of Equilibrium

We are now ready to state the main results of the paper in three propositions. Proposition 1 describes how equilibrium asset prices are linked to inflation. Proposition 2 concerns the effect of inflation on equilibrium welfare. Finally, Proposition 3 describes the effect of inflation on the volume of OTC trade. Detailed comparative statics of the variables summarizing the OTC market frictions,  $\lambda$  and  $f$ , are provided in the Web Appendix.

**Proposition 1.** *Recall the definition of the cutoff level of asset supply  $\bar{A}$  from Lemma 4. Then:*

**Case 1:** *If  $A \geq \bar{A}$ , then equilibrium will be in Regions 1 or 3. Therefore, for any  $\mu > \beta - 1$ ,  $\psi = \beta d$ , i.e. the CM asset price is at the fundamental value, and it is not affected by monetary policy.*

**Case 2:** *If  $A < \bar{A}$ , then:*

a) *For all  $\mu \in (\beta - 1, \mu'] \cup (\mu''', \infty)$  (Regions 1 and 3),  $\psi = \beta d$ ;*

b) *For all  $\mu \in (\mu', \mu'')$  (Region 5), and assuming  $\lambda > 0$ ,<sup>17</sup> the CM asset price exceeds the fundamental value, and it is a strictly increasing function of  $\mu$ . In terms of the equilibrium object  $q_2$  (see Definition 1), we can explicitly characterize it as:*

$$\psi = \left[ 1 + \lambda f \frac{u'(q_2) - 1}{(1 - \lambda)u'(q_2) + \lambda} \right] \beta d. \quad (13)$$

c) *For all  $\mu \in [\mu'', \mu''']$  (so that equilibrium real balances lie on the boundary of Regions 3 and 5 in Figure 1), the CM asset price is indeterminate within the range  $[\beta d, \psi(\mu'')]$ .*

**In both cases,** *the (real) OTC asset price,  $\varphi\psi_I$ , could increase or decrease with higher inflation. In terms of the equilibrium objects  $q_1$  and  $q_2$ , we can explicitly characterize it for any region as:*

$$\varphi\psi_I = \frac{\frac{1+\mu}{\beta}}{(1 - \lambda) \frac{u(q_2) - u(q_1)}{q_2 - q_1} + \lambda} \beta d. \quad (14)$$

*Proof.* See the appendix. □

If the asset is plentiful, in the sense that  $A \geq \bar{A}$ , the C-type in an OTC meeting will not be constrained by her asset holdings, in any equilibrium. Whether the extra liquidity will allow her to purchase  $q^*$  in the forthcoming DM, or not, depends on whether the equilibrium is in Region 1 or 3, which, in turn, depends on whether  $\mu$  is smaller or greater than  $\tilde{\mu}$ . In either case, however, one additional unit of the asset cannot help the agents increase their DM consumption, since the asset constraint is already not binding. As a result, the asset can never be priced

<sup>17</sup> The assumption that  $f \in (0, \min\{\ell, 1 - \ell\})$  is maintained throughout. Note that Region 5 is not empty if  $\lambda = 0$ , even though the liquidity premium is zero in that limiting case.

higher than its fundamental value,  $\psi = \beta d$ , and this price will not be affected by changes in the monetary policy (Case 1 of the proposition). This case is depicted in Figure 3.

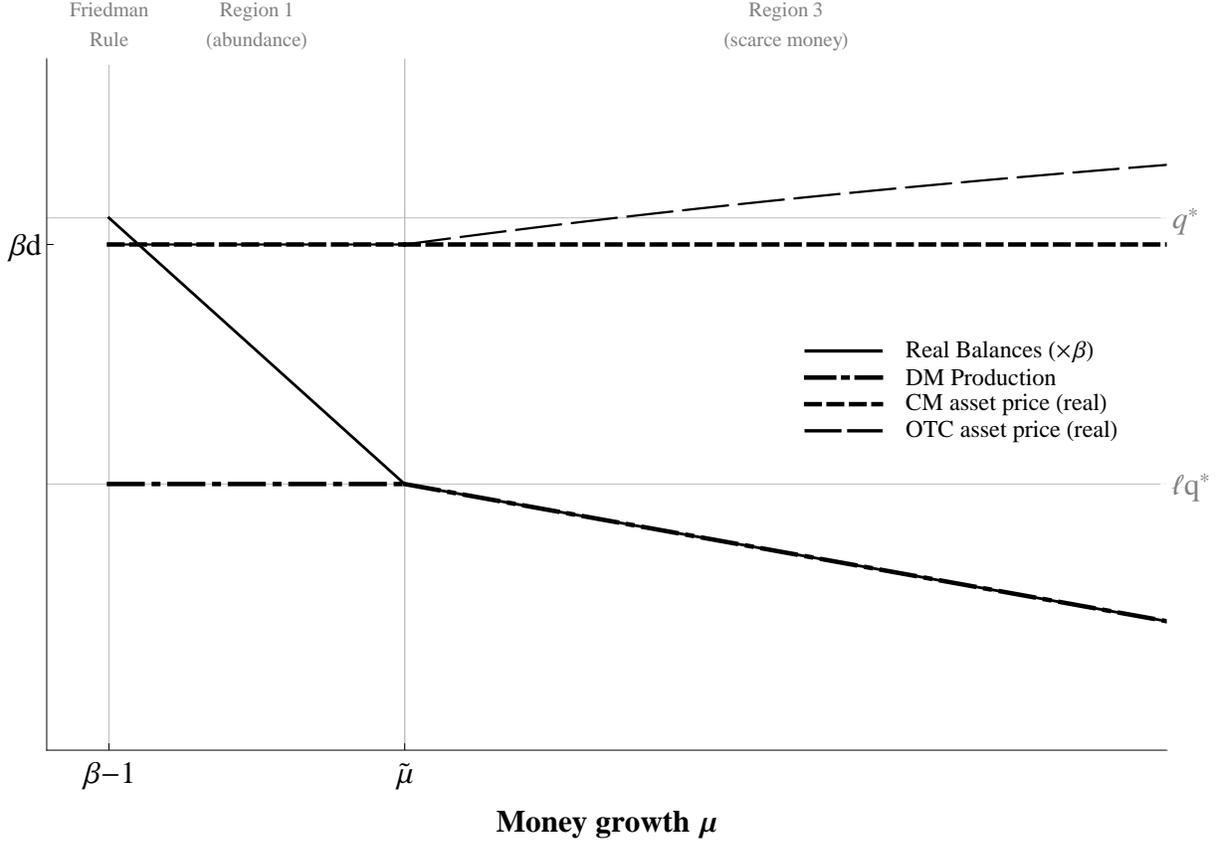


Figure 3: Effects of money growth on equilibrium variables, with  $A \geq \bar{A}$  and  $f = \ell$ .

The equilibrium properties of the asset price are richer and more interesting if the asset is relatively scarce. This case is depicted in Figure 4. With  $A < \bar{A}$ , the levels of inflation  $\mu' < \mu'' < \mu'''$ , defined in Lemma 4, become critical. For  $\mu \leq \mu'$ , equilibrium lies in Region 1, and for  $\mu > \mu'''$ , it lies in Region 3. Although for different reasons, in both of these regions, in any OTC meeting, the asset constraint does not bind, i.e. the asset is plentiful. In Region 1, the asset is abundant because agents carry a lot of their own money, so they do not need a large transfer of liquidity. In Region 3, agents carry so little money that the amount of asset  $A$  is in fact enough to purchase all of that money. In a sense, the asset is abundant because *real balances are scarce*. In either region,  $\psi = \beta d$  (part (a) of Case 2).

For  $\mu \in (\mu', \mu'')$ , equilibrium lies in Region 5. In this region the asset is scarce, in the sense that one additional unit of the asset could relax the OTC bargaining constraint, and allow the C-type to increase her DM consumption.<sup>18</sup> Hence, in the parameter space under consideration, the CM asset price carries a *liquidity premium*, which reflects the ability of a marginal increase of the asset supply to boost the agent's consumption in the DM. This result obtains even though

<sup>18</sup> Mathematically, this is represented by the second line of  $J_2^5$  in (a.7).

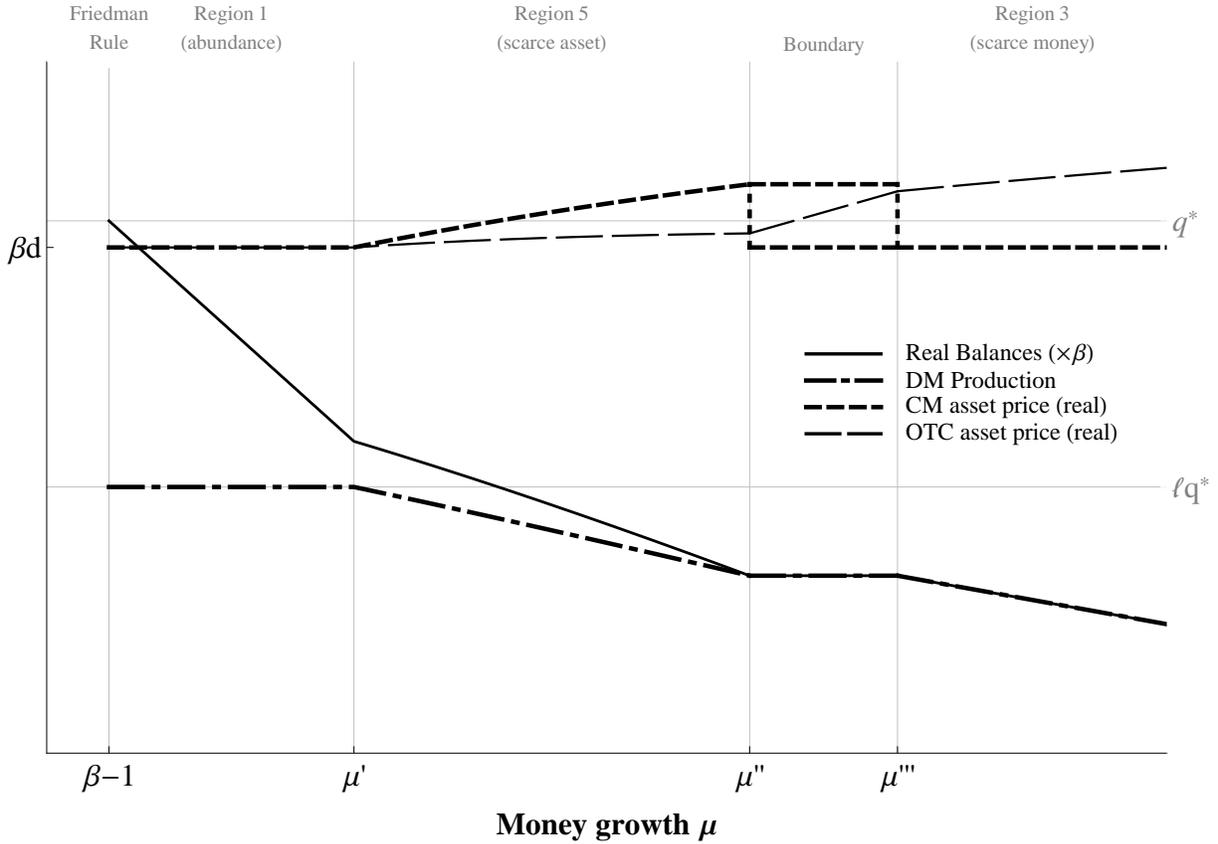


Figure 4: Effects of money growth on equilibrium variables, with  $A < \bar{A}$  and  $f = \ell$ .

the asset does not serve as a medium of exchange or collateral, nor does it help agents enter a repurchase agreement, as discussed in Lagos (2011). In our framework, the asset is sold at a premium because of its property to help agents rebalance their portfolios once the liquidity shocks have been realized.

Keeping  $A$  fixed and increasing  $\mu$  within the range  $(\mu', \mu'')$  (i.e. within Region 5) implies that each agent leaves the CM with fewer real balances. Thus, for a C-type who gets matched in the OTC, her valuation of the N-type's money is now larger. Since carrying more assets into the OTC allows the C-type to acquire more of that money, agents are willing (ex ante) to pay a price  $\psi$  that is increasing in  $\mu$  (part (b) of Case 2). The CM asset price is also positively related to the liquidity of the secondary asset market, a result which is empirically relevant (see footnote 2). To see this point, consider an increase in  $f$ , which captures the efficiency of trade in the OTC market. It is straightforward to show that  $\psi$  increases in equilibrium. First, a higher  $f$  directly increases  $\psi$  because the asset is more likely to be used in OTC trade. Second, the higher expectation of trade in the OTC market reduces the demand for real balances in the CM, in turn reducing  $q_2$ . Both effects increase  $\psi$  as can be seen in equation (13).

The results presented in Case 2, part (b) of Proposition 1 (existence of a liquidity premium which is increasing in  $\mu$ ), are in accordance with findings in Geromichalos *et al* and Lester *et al*.

However, in our model, the forces that lead to these results are quite different. In the papers mentioned above, fiat money and real assets compete as media of exchange. An increase in  $\mu$  induces agents to substitute money with the real asset, thus increasing its demand and, consequently, its price. Here, in contrast, the real asset cannot serve as a medium of exchange in the DM, but it can help agents increase their liquidity indirectly, when swapped with money in the OTC market. As  $\mu$  goes up, the agents' valuation of the asset also rises, because the service that the asset provides, *avoiding the inflation tax*, becomes more essential. Another difference with those papers is that in them, asset prices are above the fundamental even for extremely high values of  $\mu$ .<sup>19</sup> As we already explained, in this paper, the CM asset price will equal its fundamental value for high levels of inflation.

Next, consider Case 2, part (c) of the proposition, i.e. the region of monetary policies  $\mu \in [\mu'', \mu''']$ . Recall from Lemma 4 that this range represents the boundary of Regions 3 and 5 in Figure 1, and we know that, at this boundary, the demand for money exhibits a discontinuity. Therefore, any change of  $\mu$  within this range leaves equilibrium real balances unaffected. Since  $z$  does not move with  $\mu$ , and since the asset is enough to purchase exactly the optimal amount of real balances in the OTC, the buyer is happy to purchase  $A$  units of the asset in the CM, for any price between the fundamental,  $\beta d$ , and the supremum of the set  $\{\psi(\mu) : \mu \in (\mu', \mu'')\}$ . Hence,  $\psi$  is indeterminate in this region.

One of the most novel results of the paper, discussed in Proposition 1, concerns the effect of monetary policy on the (real) OTC asset price  $\varphi\psi_I$ . Unlike the CM asset price, which is typically non-decreasing in  $\mu$ , and strictly increasing for certain parameter values ( $A < \bar{A}$  and  $\mu \in (\mu', \mu'')$ ), the OTC price could increase or decrease with  $\mu$ . In principle, a higher inflation rate generates two opposing effects. On the one hand, the resulting lower value of money (represented by the term  $(1 + \mu)/\beta$  in the numerator of equation (14)) strengthens the desire of a (matched) N-type to get rid of her money. This tends to increase the OTC price. On the other hand, since agents leave the CM with less money, the C-type's valuation for her counterparty's money rises (represented by the increase from  $q_1$  to  $q_2$  in the denominator of equation (14)), so she is willing to accept a lower price for the asset.<sup>20</sup>

Which of the two forces described above dominates depends on parameter values, specifically, on  $\lambda$ ,  $\ell$ , and  $f$ , which govern the elasticity of real balances with respect to inflation. The real OTC asset price is most likely to be decreasing in  $\mu$  when  $\lambda$  is low,  $\ell$  is low, and  $f$  is high (keeping in mind, however, the physical constraint  $f \leq \min\{\ell, 1 - \ell\}$ ). When  $\lambda$  is low, the bargaining outcome is closer to the C-type's outside option, which becomes worse for higher  $\mu$ , because the C-type is more desperate to obtain extra money. When  $\ell$  is low, consumption opportunities are rare, and the incentive to hold money ex-ante is very sensitive to inflation. In

<sup>19</sup> In some cases, like in Geromichalos *et al*, this is true because for very high values of  $\mu$  monetary equilibrium collapses, since agents prefer to hold only the real asset.

<sup>20</sup> As  $q_1 \rightarrow q_2 \rightarrow q^*$ , the ratio  $[u(q_2) - u(q_1)]/(q_2 - q_1)$  converges to  $u'(q^*) \equiv 1$ . Because  $u$  is concave, this ratio is greater than 1 whenever  $q_1 < q_2 \leq q^*$ .

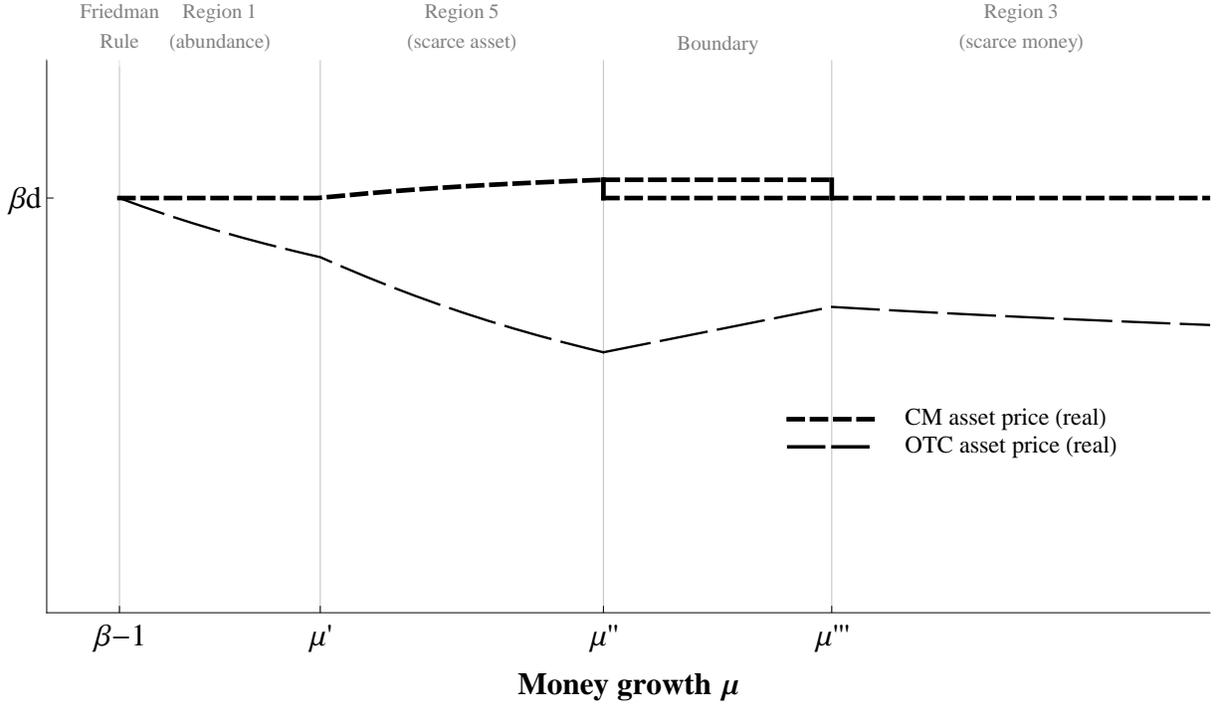


Figure 5: Effects of money growth on asset prices, with  $A < \bar{A}$ ,  $f = \ell$ , and  $\lambda, \ell$  low. Unlike in Figure 4, the OTC asset price is decreasing in inflation, except from the boundary region.

a similar fashion, when  $f$  is high, agents know that they can obtain more money as needed in the OTC market, therefore money demand is very elastic. In both cases, real balances fall very quickly as  $\mu$  increases, which makes the C-type willing to accept a lower price for the asset as she needs to acquire more money in the OTC. Figure 5 depicts a case of low  $\lambda$ , low  $\ell$ , and perfect matching  $f = \ell$ , as an example of how the OTC asset price could be decreasing with inflation.

The next proposition describes equilibrium production in the DM, which in this model is a sufficient statistic for welfare.<sup>21</sup>

**Proposition 2.** Define the average DM production as  $q_{DM} \equiv (\ell - f)q_1 + fq_2$ .

**Case 1:** If  $A \geq \bar{A}$ , then:

- a) For all  $\mu \leq \tilde{\mu}$ ,  $q_{DM}$  is strictly decreasing in  $\mu$ , unless  $f = \ell$ , in which case it is unaffected by  $\mu$ ;
- b) For all  $\mu > \tilde{\mu}$ ,  $q_{DM}$  is strictly decreasing in  $\mu$ .

**Case 2:** If  $A < \bar{A}$ , then:

- a) For all  $\mu \leq \mu'$ ,  $q_{DM}$  is strictly decreasing in  $\mu$ , unless  $f = \ell$ , in which case it is unaffected by  $\mu$ ;
- b) For all  $\mu \in (\mu', \mu'') \cup (\mu''', \infty)$ ,  $q_{DM}$  is strictly decreasing in  $\mu$ ;
- c) For all  $\mu \in [\mu'', \mu''']$ ,  $q_{DM}$  is unaffected by  $\mu$ .

*Proof.* See the appendix. □

<sup>21</sup> This result is standard in monetary-search theory. For a detailed proof, see Rocheteau and Wright (2005).

Consider first the case of a plentiful asset,  $A \geq \bar{A}$ . The equilibrium real balances and the quantity  $q_1$  are strictly decreasing in  $\mu$  throughout the range of monetary policies. However,  $q_2$ , the amount of good produced in the DM when the buyer was matched in the preceding OTC, is constant for  $\mu \leq \tilde{\mu}$ . This is true because, for this range of inflations, each agent carries real balances that allow her to purchase  $q^*/2$ , or more, in the DM (on her own). Since the asset is plentiful, in any OTC meeting, the C-type buys from the N-type the exact amount of real balances that allows her to purchase  $q^*$  in the DM. Hence, if all C-types match in the OTC, i.e.  $f = \ell$ , we have  $q_{DM} = \ell q_2$ , which is not affected by  $\mu$  (part (a) of Case 1). Finally, if  $\mu > \tilde{\mu}$ , we are in Region 3, and by Definition 1,  $q_2 = 2q_1$ . In this case,  $q_{DM} = (\ell + f)q_1$ , which is decreasing in  $\mu$  (part (b) of Case 1). These results are depicted in Figure 3.

In the scarce asset case, a higher inflation reduces the equilibrium real balances, with the exception of the region  $[\mu'', \mu''']$ . As we know from previous discussion, an increase in  $\mu$  within this region leaves real balances unaffected, and, hence, for any  $\mu$  in that region both  $q_1$  and  $q_2$  remain constant, and so does the DM production  $q_{DM}$  (part (c) of Case 2). For any  $\mu \notin [\mu'', \mu''']$ , the decreasing real balances imply that  $q_1$  is necessarily decreasing. The quantity  $q_2$  is also decreasing, with the exception of the region  $(\beta - 1, \mu']$ . Hence, for any  $\mu \notin [\mu'', \mu''']$ ,  $q_{DM} = (\ell - f)q_1 + f q_2$ , is decreasing in  $\mu$ , with a potential exception in region  $(\beta - 1, \mu']$ , when  $f = \ell$ . In that case, we have  $q_{DM} = \ell q_2$ , which is unaffected by  $\mu$  (parts (a) and (b) of Case 2). These results can be observed in Figure 4.

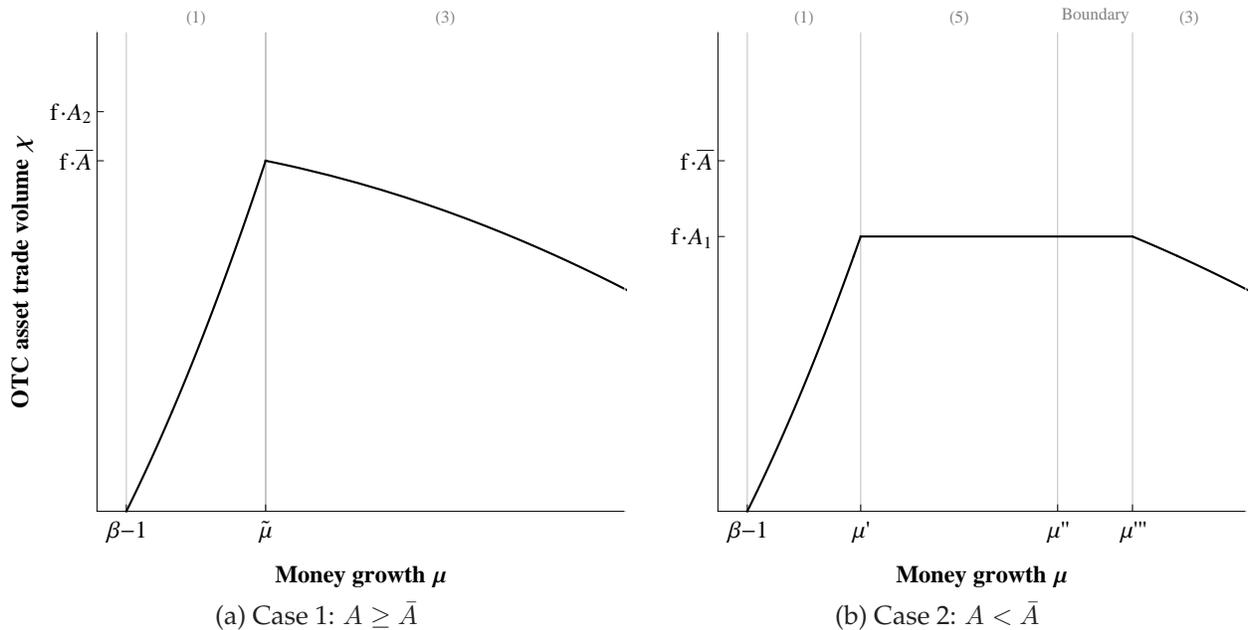


Figure 6: Trade volume in the OTC market.

Finally, consider the effect of inflation on the volume of OTC trade. Proposition 3 states the relevant results.

**Proposition 3.** *The real volume of OTC trade is given by  $f\chi$ . We have the following cases:*

**Case 1:** *If  $A \geq \bar{A}$ , then  $f\chi$  is strictly increasing for all  $\mu < \tilde{\mu}$ , and strictly decreasing for all  $\mu > \tilde{\mu}$ .*

**Case 2:** *If  $A < \bar{A}$ , then  $f\chi$  is strictly increasing for all  $\mu < \mu'$ , constant for all  $\mu \in [\mu', \mu''']$ , and strictly decreasing for all  $\mu > \mu'''$ .*

*Proof.* See the appendix. □

Figure 6 graphs the volume of OTC trade against  $\mu$ , for two levels of asset supply,  $A_2 > \bar{A}$  and  $A_1 < \bar{A}$ . In both cases,  $f\chi$  is hump-shaped, which is a very intuitive result. When inflation is low, agents leave the CM with a lot of money, thus they need small transfers of liquidity in the OTC, implying that only a small volume of assets will be traded. At the Friedman rule, the OTC market practically closes down because the role of the asset, as a device that helps agents avoid the inflation tax, is completely diminished. As  $\mu$  goes up, the role of the OTC is upgraded, and the volume of trade in that market rises as well. Finally, beyond a certain level of  $\mu$ , the trade volume declines, not because the need of agents to avoid the inflation tax is not strong enough (in fact, it is very strong), but because there is not enough liquidity in the OTC to facilitate asset trade (money is scarce). The only difference between the two panels of Figure 6 is that, when  $A < \bar{A}$ ,  $f\chi$  forms a plateau in the region  $[\mu', \mu''']$ . This result stems from the fact that, for  $A < \bar{A}$  and  $\mu \in [\mu', \mu''']$ , the asset constraint is binding in the OTC, placing an upper bound on the amount of assets that can be traded.

## 4.2 Secondary Asset Market Frictions and Welfare

In this section, we compare our model with a benchmark model in which the secondary asset market is perfectly competitive, and we provide a sufficient condition (an upper bound on inflation) under which equilibrium welfare will be higher in the model where the secondary asset market is OTC. The equilibrium properties of the benchmark model are derived in detail in the Web Appendix. Since in the model with a competitive secondary market all C-types get a chance to trade in that market, in order to make the two models comparable, we assume that in the OTC market  $f = \ell$  (which requires  $\ell \leq 1/2 \leq 1 - \ell$ , i.e. there are enough N-types for every C-type to match with).

**Proposition 4.** *Assume that  $f = \ell$ ,  $\ell \in (0, 1/2]$ , and  $\lambda < 1$ . If  $\mu \leq \mu'$  given that  $A < \bar{A}$ , or if  $\mu \leq \tilde{\mu}$  given that  $A \geq \bar{A}$ , then welfare is higher in the model with the OTC secondary asset market compared to the model with the competitive secondary asset market.*

Seen from a broader perspective, the fact that the model with a competitive secondary asset market does not deliver the efficient outcome means that the First Welfare Theorem does not

hold in our setting, and this is true because of the presence of an externality. More precisely, in our model, equilibrium welfare depends on how much of the good buyers can purchase in the DM, which, in turn, depends on the value that the economy assigns to the liquid asset, i.e. the equilibrium price of fiat money in the CM. Importantly, buyers choose their demand for money in that market without knowing whether they will actually get to use it, and understanding that they will have an additional opportunity to acquire liquidity in the secondary asset market. When a buyer chooses to hold one additional dollar, not only does she increase her own insurance against the liquidity shock, but she also provides insurance to the other agents who could acquire that dollar in the secondary market. However, the representative buyer does not take this effect into account when choosing her demand for money.

Given our setup, the secondary asset market allows agents to avoid the inflation tax, which may depress the real value of money. With an OTC secondary market, buyers can only obtain extra liquidity at a cost that depends on their bargaining power. If inflation is not too high, they will prefer to carry their own money from the CM rather than obtain it in the OTC. As a result, agents' real balances will be close enough to the first-best level so that C-types who match in the OTC can in fact purchase  $q^*$  in the DM. These results are depicted in Figure 7.

It should be pointed out that we have assumed  $f = \ell$ , in order to make the two specifications of the model comparable and obtain a sharp characterization of welfare in Proposition 4. However, setting  $f = \ell$  (i.e. no matching frictions in the OTC) is not a necessary condition for our result. In the Web Appendix, we demonstrate that welfare can be higher in the model with OTC secondary trade, even if  $f$  is significantly smaller than  $\ell$ . Moreover, low inflation is sufficient but not necessary for the proposition, as Figure 7 also illustrates.

Our result described in Proposition 4 belongs to a literature showing that under limited commitment, pairwise trading can achieve better outcomes than competitive trading. The main reference is Hu, Kennan, and Wallace (2009), where pairwise meetings in the trade for specialized goods (equivalent to our DM) can achieve first-best while centralized trade with competitive pricing cannot. We, by contrast, focus on trade in the financial market preceding this goods market after agents have already made second-best portfolio decisions. We show that a search-and-bargaining friction in this market can mitigate the original friction, to the extent where our economy can (for the right parameters and inflation not too high) achieve first-best outcomes even after second-best decision making.

The result is also similar in spirit to Berentsen, Huber, and Marchesiani (2014), who consider an environment similar to ours but assume that agents rebalance their positions in a competitive secondary asset market. Like in our paper, when the representative agent chooses her portfolio in the CM, she does not take into account the externality of providing insurance to other market participants. The authors carry out a comparative statics exercise and show that restricting access to the secondary asset market may internalize this externality and improve

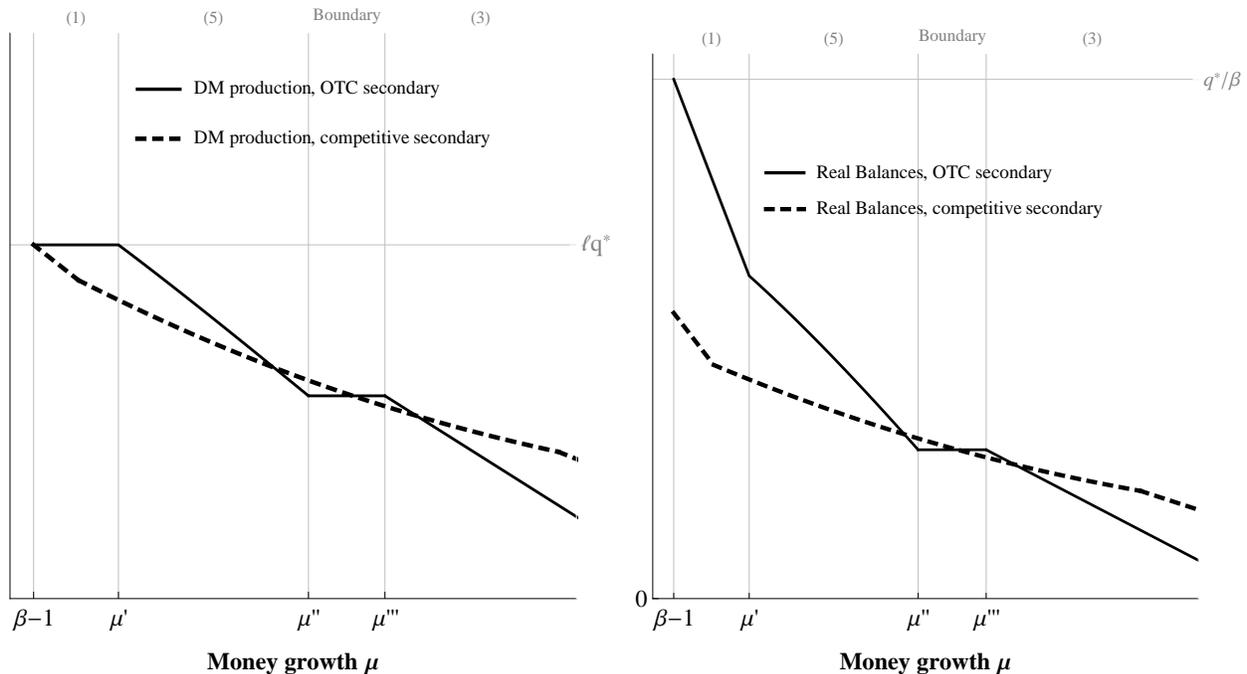


Figure 7: Comparison of  $q_{DM}$  and  $z$  in the two models. Here  $A < \bar{A}$ .

welfare (this result obtains for high levels of inflation).<sup>22</sup> We conduct a different experiment: we compare equilibrium outcomes between two alternative structures of the secondary asset market and show that welfare can be higher under the frictional specification. Another difference is that the result of Berentsen *et al* (2014) obtains if inflation is sufficiently high, whereas ours obtains more generally.<sup>23</sup>

## 5 Conclusions

We revisit a traditional question in monetary economics, the relationship between asset prices and monetary policy. A real asset in fixed supply is issued in a centralized market, *à la* Lagos and Wright (2005), where agents also choose their money holdings. After the uncertainty regarding consumption in the decentralized market has been resolved, agents may visit a secondary asset market where they can rebalance their positions depending on their liquidity

<sup>22</sup> Chiu and Meh (2011) also obtain a similar result, although, in their analysis, there is only one asset (fiat money), and intermediation, i.e., the channeling of funds to agents with a liquidity need takes place through a competitive banking sector. For intermediate levels of inflation agents borrow funds, but these funds are used to finance trades of relatively low value. The authors show that the welfare in an economy with banking can be lower than the one in an economy where banking is banned. However, in their analysis, borrowing through banks incurs an intermediation cost, which is not the case in our paper.

<sup>23</sup> In recent work, Auster (2014) obtains a similar result, albeit in a completely different framework. The author studies bilateral trade under ambiguity, and shows that ambiguity over the seller's willingness to accept an offer can have a positive effect on the volume of trade. Hence, her model suggests that certain assets may be optimally traded OTC rather than on traditional exchanges, and that opacity may be essential to sustain this type of trade.

needs. We model this secondary asset market in the style of the OTC markets of Duffie, Gârleanu, and Pedersen (2005), in which trade is bilateral and agents bargain over the price. This allows us to link asset pricing in OTC markets to monetary policy.

We show that, if the supply of the asset is sufficiently small, the issue price of the asset can include a liquidity premium, reflecting the fact that the asset can help agents avoid the inflation tax. This premium is increasing in inflation, because the service that the asset provides is more essential when inflation is high. The effect of inflation on the secondary market asset price is ambiguous, and depends on agents' bargaining power in the OTC market, and on how sensitive real balances are to inflation. We also consider the effect of inflation on OTC trade. When inflation is low, agents have little need for additional real balances, and when inflation is high, agents have few real balances to trade. Although for different reasons, asset trade is low in both extremes, and hump-shaped as a function of inflation overall. Finally, we compare our model with a benchmark model where the secondary asset market is competitive. We show that, in low inflation regimes, search frictions in the secondary asset market induce agents to choose real balances more efficiently, thus leading to greater equilibrium welfare.

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## A Appendix

### A.1 Optimal Behavior of Buyers

#### A.1.1 Regions of the state space

We will examine the buyer’s optimal choice of  $(\hat{m}, \hat{a})$  for any possible money and asset prices, and for any given beliefs about other buyers’ money and asset holdings. We focus on prices that satisfy  $\varphi > \beta\hat{\varphi}$ , since we know that this will be always true in steady state equilibria with  $\mu > \beta - 1$ . Also, the asset price has to satisfy  $\psi \geq \beta d$ , since violation of this condition would generate an infinite demand for the asset. The optimal behavior of the buyer is described in detail in Lemmas 5 and 6 below. However, since the results of these lemmas are crucial for the analysis, we provide here a non-technical description of the buyer’s optimal portfolio choice.

The objective function of the buyer depends on the terms  $\chi, \psi_I, \tilde{\chi}$ , and  $\tilde{\psi}_I$ , which, in turn, depend on the bargaining protocol in the OTC market. Given the buyer’s beliefs  $(\tilde{m}, \tilde{a})$ , she can end up in different branches of the bargaining solution, depending on her own choices of  $(\hat{m}, \hat{a})$ . In general, the domain of the objective function can be divided into five regions, arising from three questions: (i) When the C-type and the N-type pool their money in the OTC market, can they achieve the first-best in the DM? (ii) If I am a C-type, do I carry enough assets to compensate the N-type? (iii) If I am an N-type, do I expect a C-type to carry enough assets to compensate me? These regions are illustrated in Figure 8 and the derivation of the figure is described below. But first, we provide an intuitive explanation of how the OTC terms of trade vary in the different regions. For this discussion, it is important to recall the definition of the asset cutoff term  $\bar{a}(\cdot, \cdot)$  from Lemma 2.

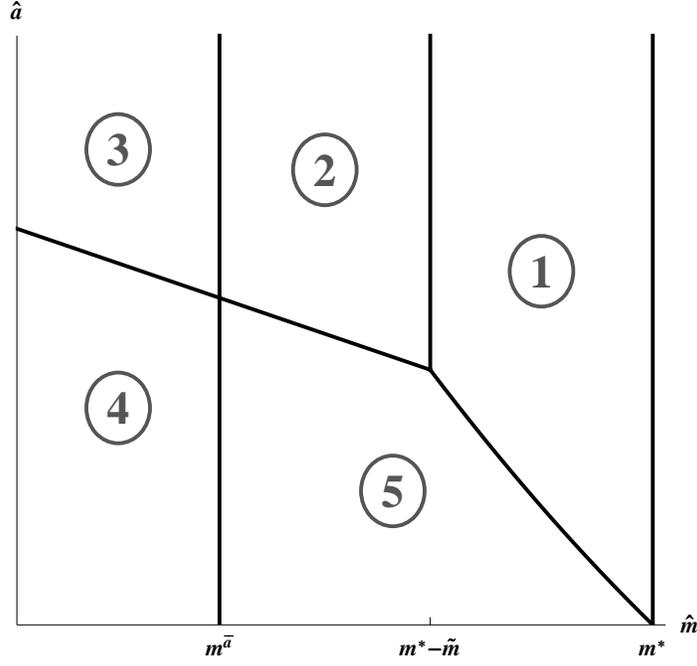


Figure 8: Regions of the individual choice problem.

1.  $\hat{m} \in (m^* - \tilde{m}, m^*)$  and  $\hat{a} > \bar{a}(\hat{m}, \tilde{m})$ .

In this region, the money holdings of the C-type and the N-type together allow the C-type to bring the first-best  $m^*$  into the DM. If the agent is a C-type, her asset holdings are enough to compensate an N-type for her money. If the agent is an N-type, the potential counterparty may or may not carry enough assets to purchase the first best level of money,  $m^* - \tilde{m}$ , but that is a level effect on  $J(\hat{m}, \hat{a})$  and does not affect the optimal choice.<sup>24</sup>

2.  $\hat{m} < m^* - \tilde{m}$ ,  $\hat{a} > \bar{a}(\hat{m}, \tilde{m})$ , but  $\tilde{a} < \bar{a}(\tilde{m}, \hat{m})$ .

Here there is not enough money in an OTC match to allow the C-type to bring  $m^*$  into the DM. If a C-type, the agent carries enough assets to buy all the money of the N-type, but if an N-type, the agent does not expect the C-type counterparty to carry enough assets to buy all of the agent's money.

3.  $\hat{m} < m^* - \tilde{m}$ ,  $\hat{a} > \bar{a}(\hat{m}, \tilde{m})$ , and  $\tilde{a} > \bar{a}(\tilde{m}, \hat{m})$ .

There is not enough money in an OTC match to allow the C-type to bring  $m^*$  into the DM. In an OTC match, the agent expects all of the money of the N-type to be traded for less than all of the assets of the C-type (regardless of whether the buyer in question is the C or the N-type).

4.  $\hat{m} < m^* - \tilde{m}$ ,  $\hat{a} < \bar{a}(\hat{m}, \tilde{m})$ , but  $\tilde{a} > \bar{a}(\tilde{m}, \hat{m})$ .

<sup>24</sup> Since here the objective is to describe the buyer's optimal behavior, we focus on how different choices of  $(\hat{m}, \hat{a})$  lead to different branches of the OTC bargaining protocol. In Region 1, the buyer is not certain whether her C-type counterparty is asset constrained or not, but she also does not care. What determines Region 1 is that within it the buyer's money holdings never affect the terms of trade when she is an N-type. For more details, see the appendix.

There is not enough money in an OTC match to allow the C-type to bring  $m^*$  into the DM. If a C-type, the agent does not carry enough assets to buy all the money of the N-type, but if an N-type, the agent expects the C-type counterparty to carry enough assets to buy all her money.

5.  $\hat{a} < \bar{a}(\hat{m}, \tilde{m})$ , and either  $\tilde{a} < \bar{a}(\tilde{m}, \hat{m})$  or  $\hat{m} \in (m^* - \tilde{m}, m^*)$ .

If a C-type, the agent does not carry enough assets to buy all the money of the N-type. If an N-type, the agent expects not to give away all of her money, either because the C-type counterparty does not carry enough assets to afford it, or because she does not need all of that money. This distinction does not affect the buyer's optimal choice.

The various regions of Figure 8 represent different branches of the OTC bargaining solution. On the horizontal axis of that figure we measure the buyer's money holdings in the OTC,  $\hat{m}$ , and on the vertical axis we measure her real asset holdings,  $\hat{a}$ . Due to our assumption that  $\mu > \beta - 1$ , the money holdings of the buyer will never satisfy  $\hat{m} \geq m^*$ . Also, in what follows, we will rule out the case in which  $\tilde{m} \geq m^*$ .<sup>25</sup>

The relevant regions arise from the answers to the following three questions:

Question 1: The first question that needs to be answered is whether the sum of the money holdings of a C-type and an N-type within an OTC match, allows the C-type to achieve the first-best in the DM. The answer to this question is affirmative, if and only if the buyer's money holdings,  $\hat{m}$ , exceeds the cutoff level  $m^* - \tilde{m}$ , depicted in Figure 8.

Question 2: The second question that we need to ask is whether the buyer, in the event of being a C-type, carries enough assets to compensate the N-type for the transfer of money that takes place. There are two cases:

Case a: If  $\hat{m} \geq m^* - \tilde{m}$ , we know from Lemma 2, that the amount of assets that C-type needs to carry in order to buy the first best amount of money is given by

$$\bar{a} = \frac{(1 - \lambda) [u(q^*) - u(\beta \hat{\varphi} \hat{m})] + \lambda \beta \hat{\varphi} (m^* - \hat{m})}{\beta d}.$$

The derivative of this term with respect to  $\hat{m}$  can be shown to be equal to

$$\frac{\partial \bar{a}}{\partial \hat{m}} = -\frac{\hat{\varphi}}{d} [\lambda + (1 - \lambda) u'(\beta \hat{\varphi} \hat{m})] < 0.$$

Case b: If  $\hat{m} \leq m^* - \tilde{m}$ , we know from Lemma 2, that the N-type should hand over to the C-type all her money. The amount of assets that the C-type needs to carry in order to be able to

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<sup>25</sup> Strictly speaking, the buyer may not believe that other agents are rational and carry into the OTC an amount of money no greater than  $m^*$ . However, we know that this case will never be relevant in equilibrium. Hence, since we already have to deal with many cases of asset holdings, we choose to ignore this rather trivial region.

afford this amount of money is given by

$$\bar{a} = \frac{(1 - \lambda) \{u[\beta\hat{\varphi}(\hat{m} + \tilde{m})] - u(\beta\hat{\varphi}\hat{m})\} + \lambda\beta\hat{\varphi}\tilde{m}}{\beta d},$$

and in this case, we have

$$\frac{\partial \bar{a}}{\partial \hat{m}} = \frac{(1 - \lambda)\hat{\varphi}}{\beta d} \{u'[\beta\hat{\varphi}(\hat{m} + \tilde{m})] - u'(\beta\hat{\varphi}\hat{m})\},$$

which is, clearly, also strictly negative.

Summing up, the C-type carries enough assets to compensate the N-type for the optimal transfer of money, if and only if  $\hat{a}$  lies above the curve  $\bar{a}(\hat{m}, \tilde{m})$  in Figure 8. This curve is strictly decreasing for the whole relevant domain of  $\hat{m}$ , and it is characterized by a kink at  $\hat{m} = m^* - \tilde{m}$ .<sup>26</sup> Of course, as  $\hat{m} \rightarrow m^*$ ,  $\bar{a} = 0$  regardless of  $\tilde{m}$ , since the C-type can afford  $q^*$  on her own and does not need to buy any of the N-type's money.

Question 3: The third question that needs to be answered is whether a buyer who turns out to be an N-type, expects the C-type counterparty to carry enough assets to compensate her for her money. In other words, we are interested in whether  $\tilde{a} \geq \bar{a}(\tilde{m}, \hat{m})$ . Again we distinguish two cases:

Case a: If  $\hat{m} \geq m^* - \tilde{m}$ , and the buyer brings one additional unit of money, the amount of assets that the N-type needs in order to buy that money does not change. This is because the money that changes hands in this case is given by  $m^* - \tilde{m}$ , which is not affected by  $\hat{m}$ .

Case b: If  $\hat{m} \leq m^* - \tilde{m}$ , the N-type buyer's money holdings become relevant for the answer to Question 3. Depending on the beliefs  $(\tilde{m}, \tilde{a})$ , there will exist a threshold of money holdings, denoted by  $m^{\tilde{a}}$  in Figure 8, such that  $\tilde{a} \geq \bar{a}(\tilde{m}, \hat{m})$  if and only if  $\hat{m} \leq m^{\tilde{a}}$ . This threshold is increasing in  $\tilde{a}$  and decreasing in  $\tilde{m}$ . The logic is as follows. For a given  $\tilde{m}$ , the higher  $\tilde{a}$  is, the more money I will need to bring in order to get into the region where my counterparty cannot afford to buy my money. On the other hand, for a given  $\tilde{a}$ , as  $\tilde{m}$  goes up, the C-type counterparty needs less of my money, because she carries more of her own.

How far can the threshold  $m^{\tilde{a}}$  go? Recall that in the region where  $\hat{m} \geq m^* - \tilde{m}$ , the buyer's money holdings do not affect the term  $\bar{a}(\tilde{m}, \hat{m})$ . Hence,  $m^{\tilde{a}}$  can never lie on the right of the point  $m^* - \tilde{m}$ . As  $\tilde{a}$  becomes large, relatively to  $\tilde{m}$ ,  $m^{\tilde{a}}$  will exactly coincide with the perpendicular line  $m^* - \tilde{m}$ . It is easy to verify that this will be the case as long as  $\tilde{a} \geq \bar{a}(\tilde{m}, m^* - \tilde{m})$ . It is also easy to see that, in this case, Region 2 in Figure 8 ceases to exist.

Notice that the plot in Figure 8 is based on the implicit assumption that  $\tilde{a} < \bar{a}(\tilde{m}, m^* - \tilde{m})$ , so that Region 2 exists. However, the verbal description of the five regions on page 29 is valid for any set of beliefs  $(\tilde{m}, \tilde{a})$ . For example, in the description of Region 1, we state that "If the agent is

<sup>26</sup> It can be easily shown that the absolute value of the slope of  $\bar{a}$  is lower on the left side of the kink. However, this detail does not play any important role in the analysis.

an N-type, the potential counterparty may or may not carry enough assets to purchase...  $m^* - \tilde{m}$ , but... (this) does not affect the optimal choice". What we really mean is that it does not matter whether the C-type counterparty carries enough assets or not. What matters, or more precisely, what determines Region 1, is that within this region the buyer's money holdings never affect the terms of trade when she is an N-type. A similar interpretation applies for Region 5.

### A.1.2 A buyer's objective function

**Lemma 5.** *The objective function  $J : \mathbb{R}_+^2 \rightarrow \mathbb{R}$  is:*

- i. continuous everywhere;*
- ii. differentiable within each of the five regions defined above;*
- iii. strictly concave in the first argument (money) everywhere;*
- iv. concave in the second argument (asset), strictly in Regions 4 and 5;*
- v. weakly concave everywhere.*

Finally, letting  $J^i(\hat{m}, \hat{a})$ ,  $i = 1, \dots, 5$ , denote the objective function in region  $i$ , and  $J_k^i(\hat{m}, \hat{a})$ ,  $k = 1, 2$ , its derivative with respect to the  $k$ -th argument, we have:

$$J_1^1(\hat{m}, \hat{a}) = -\varphi + \beta\hat{\varphi} + \beta\hat{\varphi}(\ell - \lambda f) [u'(\beta\hat{\varphi}\hat{m}) - 1], \quad (\text{a.1})$$

$$J_1^2(\hat{m}, \hat{a}) = -\varphi + \beta\hat{\varphi} + \beta\hat{\varphi}(\ell - \lambda f) [u'(\beta\hat{\varphi}\hat{m}) - 1] + \beta\hat{\varphi}\lambda f \{u'[\beta\hat{\varphi}(\hat{m} + \tilde{m})] - 1\}, \quad (\text{a.2})$$

$$J_1^3(\hat{m}, \hat{a}) = -\varphi + \beta\hat{\varphi} + \beta\hat{\varphi}(\ell - \lambda f) [u'(\beta\hat{\varphi}\hat{m}) - 1] + \beta\hat{\varphi}f \{u'[\beta\hat{\varphi}(\hat{m} + \tilde{m})] - 1\}, \quad (\text{a.3})$$

$$J_1^4(\hat{m}, \hat{a}) = -\varphi + \beta\hat{\varphi} + \beta\hat{\varphi}\ell [u'(\beta\hat{\varphi}\hat{m}) - 1] + \beta\hat{\varphi}(1 - \lambda)f \{u'[\beta\hat{\varphi}(\hat{m} + \tilde{m})] - 1\} \\ + \beta\hat{\varphi}\lambda f \frac{u'[\beta\hat{\varphi}(\hat{m} + \hat{a}\psi_I^a(\hat{m}, \hat{a}))] - u'(\beta\hat{\varphi}\hat{m})}{(1 - \lambda)u'[\beta\hat{\varphi}(\hat{m} + \hat{a}\psi_I^a(\hat{m}, \hat{a}))] + \lambda}, \quad (\text{a.4})$$

$$J_1^5(\hat{m}, \hat{a}) = -\varphi + \beta\hat{\varphi} + \beta\hat{\varphi}\ell [u'(\beta\hat{\varphi}\hat{m}) - 1] \\ + \beta\hat{\varphi}\lambda f \frac{u'[\beta\hat{\varphi}(\hat{m} + \hat{a}\psi_I^a(\hat{m}, \hat{a}))] - u'(\beta\hat{\varphi}\hat{m})}{(1 - \lambda)u'[\beta\hat{\varphi}(\hat{m} + \hat{a}\psi_I^a(\hat{m}, \hat{a}))] + \lambda}, \quad (\text{a.5})$$

$$J_2^1(\hat{m}, \hat{a}) = J_2^2(\hat{m}, \hat{a}) = J_2^3(\hat{m}, \hat{a}) = -\psi + \beta d, \quad (\text{a.6})$$

$$J_2^4(\hat{m}, \hat{a}) = J_2^5(\hat{m}, \hat{a}) = -\psi + \beta d \left\{ 1 - f + f \frac{u'[\beta\hat{\varphi}(\hat{m} + \hat{a}\psi_I^a(\hat{m}, \hat{a}))]}{(1 - \lambda)u'[\beta\hat{\varphi}(\hat{m} + \hat{a}\psi_I^a(\hat{m}, \hat{a}))] + \lambda} \right\}, \quad (\text{a.7})$$

where  $\psi_I^a(\cdot, \cdot)$  was defined in (10).

*Proof.* Consider first the derivatives of the objective with respect to  $\hat{m}$  and  $\hat{a}$ , i.e. equations (a.1)-(a.7). To obtain these conditions we substitute the appropriate solution to the bargaining problem (depending on the region in question) into (12), and we differentiate with respect to  $\hat{m}$  or  $\hat{a}$ .

As an illustration, consider Region 2. Recall that in this region,  $\hat{m} < m^* - \tilde{m}$ ,  $\hat{a} > \bar{a}(\hat{m}, \tilde{m})$ , but  $\tilde{a} < \bar{a}(\tilde{m}, \hat{m})$ . In words, there is not enough money in an OTC match to allow the C-type to bring  $m^*$  into the DM. If a C-type, the agent carries enough assets to buy all the money of the N-type, but if an N-type, the agent does not expect her N-type counterparty to carry enough assets to buy all of her money. Based on this information, we have  $\chi = \bar{a}(\hat{m}, \tilde{m})$ ,  $\chi\psi_I = \tilde{m}$ ,  $\tilde{\chi} = \tilde{a}$ , and  $\tilde{\chi}\tilde{\psi}_I = \tilde{a}\psi_I^a(\tilde{m}, \tilde{a})$ . Substituting these terms into the objective, implies that

$$\begin{aligned} J^2(\hat{m}, \hat{a}) = & -\varphi\hat{m} + (-\psi + \beta d)\hat{a} + f \{u[\beta\hat{\varphi}(\hat{m} + \tilde{m})] - \beta d\bar{a}(\hat{m}, \tilde{m})\} \\ & + (\ell - f) u(\beta\hat{\varphi}\hat{m}) + f \{\beta\hat{\varphi}[\hat{m} - \tilde{a}\psi_I^a(\tilde{m}, \tilde{a})] + \beta d\tilde{a}\} + (1 - \ell - f)\beta\hat{\varphi}\hat{m}. \end{aligned}$$

It is now straightforward to show that  $J_1^2$  and  $J_2^2$  are given by (a.2) and (a.6), respectively. The remaining derivations follow exactly the same steps.

Notice that we can solve  $J_1^i = 0$ ,  $i = 1, \dots, 5$ , with respect to the term  $\varphi/(\beta\hat{\varphi})$ , which, in steady state equilibrium, is just one plus the nominal interest rate. This will yield the demand for money as a function of its holding cost. For future reference, it is important to highlight that the money demand is in fact continuous on the boundaries 1-2, and 1-5.<sup>27</sup> Similarly, we can solve  $J_2^i = 0$ ,  $i = 1, \dots, 5$ , with respect to  $\psi/(\beta d)$ , in order to obtain the demand for the real asset. It can be easily verified that this function is continuous on the boundaries 1-2, 2-5, 2-3, and 4-5.

We now prove the five properties of  $J$  stated in Lemma 5.

i. The solution to the OTC bargaining problem is continuous, as can be seen from the proof of Lemma 2. One of the three constraints  $\chi\psi_I \leq \tilde{m}$ ,  $\chi\psi_I \leq m^* - m$ , and  $\chi \leq a$  must bind, together with equation (a.9). Each of these is linear in the choice variables. Therefore,  $J$  is continuous.

ii. As above, one of the constraints must bind together with equation (a.9). Each of these is differentiable in the choice variables, and within a region of  $J$ , the binding constraint does not switch. Furthermore,  $J$  is differentiable on those boundaries where both FOCs are continuous (see above).

iii. As  $J$  is continuous everywhere and differentiable within each region,  $J_1$  is defined everywhere except at a finite number of boundary crossings. We need to show that  $J_1$  is decreasing as a function of  $\hat{m}$  within each region, and that  $J_{1-} \geq J_{1+}$  on each boundary, where “-” denotes the left derivative and “+” denotes the right derivative.

That  $J_1$  is strictly decreasing in  $\hat{m}$  within Regions 1-3 follows immediately from equations (a.1)-

<sup>27</sup> The money demand is also continuous on the boundaries of the Regions 1-3 and 4-5 if  $\tilde{a} \geq \bar{a}(\tilde{m}, m^* - \tilde{m})$ , in which case Region 2 does not exist.

(a.3), and the fact that  $u'$  is strictly decreasing. In Regions 4 and 5, showing that  $J_1$  is decreasing in  $\hat{m}$  is less obvious. To simplify notation, define  $x \equiv \beta\hat{\varphi}\hat{m}$  and  $y(x) \equiv \beta\hat{\varphi}\hat{a}\psi_I^a(\hat{m}, \hat{a})$ . In Region 5 (where  $x + y < q^*$ ), we have

$$J_1^5 = -\varphi + \beta\hat{\varphi} + \beta\hat{\varphi}\ell[u'(x) - 1] + \beta\hat{\varphi}\lambda f \frac{u'[x + y(x)] - u'(x)}{(1 - \lambda)u'[x + y(x)] + \lambda}.$$

Since  $y(x)$  satisfies (10), applying total differentiation in this equation yields

$$\frac{dy}{dx} = (1 - \lambda) \frac{u'(x) - u'[x + y(x)]}{(1 - \lambda)u'[x + y(x)] + \lambda}.$$

Consequently,

$$\begin{aligned} \frac{\partial J_1^5}{\partial x} &= \frac{\beta\hat{\varphi}}{[(1 - \lambda)u'(x + y) + \lambda]^3} \left\{ f\lambda [(1 - \lambda)u'(x) + \lambda]^2 u''(x + y) \right. \\ &\quad \left. + [(\ell - f)\lambda + \ell(1 - \lambda)u'(x + y)] [(1 - \lambda)u'(x + y) + \lambda]^2 u''(x) \right\}. \end{aligned}$$

Since  $u''(\cdot) < 0$ , the entire term  $\partial J_1^5 / \partial x < 0$ . In Region 4, the only addition is a term involving  $u'(\cdot)$ , which is clearly decreasing too. Hence,  $J_1^4$  is decreasing in  $\hat{m}$  as well.

As we discussed above,  $J_1$  is continuous across all the boundaries of the various regions, except the boundaries 2-3, 3-4, 4-5, 2-5, and the crossing 2-4. With some algebra, one can check that  $J_1^2 < J_1^3$ ,  $J_1^3 < J_1^4$ ,  $J_1^4 < J_1^5$ , and  $J_1^2 < J_1^5$ , across the respective boundaries. Also,  $J_1^3 > J_1^5$  at the crossing 2-3-4-5, establishing the chain  $J_1^2 < J_1^5 < J_1^3 < J_1^4$  at this crossing. Consequently,  $J$  is concave in  $\hat{m}$  throughout.

iv. As  $J$  is continuous everywhere and differentiable within each region,  $J_2$  is defined everywhere except at a finite number of boundary crossings. We need to show that  $J_2$  is decreasing as a function of  $\hat{a}$  within each region (strictly, in Regions 4 and 5), and that  $J_{2-} \geq J_{2+}$  on each boundary, where “-” denotes the left derivative and “+” denotes the right derivative.

In Regions 1-3,  $J_2^i$  is constant, hence weakly concave. We now show that  $J_2^i$  is strictly decreasing in  $\hat{a}$  within Regions 4 and 5. Applying total differentiation in equation (10), yields

$$\frac{\partial [a\psi_I^a(m, a)]}{\partial a} = \frac{d}{\hat{\varphi}} \frac{1}{(1 - \lambda)u'[\beta\hat{\varphi}(m + a\psi_I^a(m, a))] + \lambda}.$$

Since this expression is clearly positive, and  $u'$  is strictly decreasing, it follows that  $\partial J_2^i / \partial \hat{a} < 0$ , for  $i = 4, 5$ .

Next, using the definitions of the regions, one can see that  $J_2$  is continuous across the bound-

ary 2-6, but not the boundaries 3-6 or 4-5. The term

$$\frac{u' [\beta\hat{\varphi}(\hat{m} + \hat{a}\psi_I^a(\hat{m}, \hat{a}))]}{(1 - \lambda)u' [\beta\hat{\varphi}(\hat{m} + \hat{a}\psi_I^a(\hat{m}, \hat{a}))] + \lambda}$$

is greater than 1 in Regions 5 and 6, because

$$\hat{m} + \hat{a}\psi_I^a(\hat{m}, \hat{a}) < \min\{\hat{m} + \tilde{m}, m^*\}.$$

and therefore  $u'(\cdot) > 1$ .

v. We need to show that  $J_2$  is non-increasing as a function of  $\hat{m}$  within each region, and across boundaries. First,  $J_2$  depends on  $\hat{m}$  only in Regions 5 and 6. There, the term  $\zeta \equiv \beta\hat{\varphi}[\hat{m} + \hat{a}\psi_I^a(\hat{m}, \hat{a})]$  is strictly increasing in  $\hat{m}$ , therefore  $u'(\zeta)$  is strictly decreasing, and so is  $u'(\zeta)[(1 - \lambda)u'(\zeta) + \lambda]^{-1}$ .

Now, the only boundaries where  $J_2$  is not a continuous function of  $\hat{m}$  are the boundaries of Regions 4 and 5, and 3 and 6, which are downward sloping in  $(\hat{m}, \hat{a})$ -space. On these boundaries,  $J_{2-} > J_{2+}$  (see part (iv.)). This is sufficient because an infinitesimal increase in  $\hat{m}$  has the same effect as an infinitesimal increase in  $\hat{a}$  (the definition of  $J_{2+}$ ), and vice versa, when the boundaries are downward sloping.

We conclude that  $J_2$  is weakly decreasing as a function of  $\hat{m}$ , therefore  $J$  is submodular (money and asset are strategic substitutes). As  $J$  is also weakly concave in each argument, it is weakly concave overall.  $\square$

The next lemma builds on Lemma 5 and describes in detail the optimal behavior of the representative buyer.

**Lemma 6.** *Taking prices,  $(\varphi, \hat{\varphi}, \psi)$ , and beliefs,  $(\tilde{m}, \tilde{a})$ , as given, the optimal choice of the representative buyer,  $(\hat{m}, \hat{a})$ , satisfies:*

- a) *If the optimal choice is strictly within any region, or on the boundary of Region 1 with any other region, it satisfies  $\nabla J(\hat{m}, \hat{a}) = \mathbf{0}$ .*
- b) *If  $\varphi > \beta\hat{\varphi}$  and  $\psi = \beta d$ , the optimal  $\hat{m}$  is unique, and any  $\hat{a}$  is optimal as long as  $(\hat{m}, \hat{a})$  is in Regions 1, 2, or 3 (or on their boundaries).*
- c) *If  $\varphi > \beta\hat{\varphi}$  and  $\psi > \beta d$ , the optimal choice is unique, and it lies in Regions 4 or 5 or on their boundaries, except the boundary of Regions 1 and 5.*

*Proof.* a) The fact that  $\nabla J(\hat{m}, \hat{a}) = \mathbf{0}$ , follows from the fact that  $J$  is weakly concave overall and differentiable within each region. So if the optimal choice  $(\hat{m}, \hat{a})$  is within a region, the first-order conditions must hold.

b) The fact that  $\psi = \beta d$  rules out Regions 4 and 5. Notice from (a.7) that for any  $(\hat{m}, \hat{a})$  in the interior of these regions,  $\psi = \beta d$  implies  $J_2^i > 0$ , for  $i = 4, 5$ . In Regions 1-3, money demand is strictly decreasing, so the  $\hat{m}$  satisfying  $\varphi > \beta \hat{\varphi}$  is unique. But any  $\hat{a}$  in Regions 1-3 satisfies  $J_2^i = 0$ ,  $i = 1, 2, 3$ .

c) The fact that  $\psi > \beta d$  rules out the interior of Regions 1-3 or the boundary 1-5. Notice from (a.6) that for any  $(\hat{m}, \hat{a})$  in the regions in question,  $\psi > \beta d$  implies  $J_2^i < 0$  for  $i = 1, 2, 3$ .  $\square$

Let us highlight the most important properties of the buyer's choice of asset holdings. If the asset price is at the fundamental, i.e.  $\psi = \beta d$ , the cost of carrying the asset is zero and, therefore, it would be suboptimal for the buyer to be in a region where her assets would not allow her to afford the optimal quantity of money, when a C-type. As a result, when  $\psi = \beta d$  the buyer never chooses a portfolio in the interior of Regions 4 and 5. If  $\psi > \beta d$ , carrying the real asset is costly. The optimal choice of the buyer is characterized by the first-order conditions and, graphically, it lies within Regions 4 or 5. For any given prices, the optimal choice of money is uniquely characterized by the first-order condition with respect to  $\hat{m}$ , described in detail below in Section A.1.3.

### A.1.3 Description of money demand

The money demand,  $D_m$ , is plotted in Figure 9 against the ratio  $\varphi/(\beta \hat{\varphi})$ , which captures the cost of holding money. In this graph, the level of asset holdings is kept fixed at the values  $\hat{a}_1$  and  $\hat{a}_2$ ,  $\hat{a}_2 > \hat{a}_1$ . These values are indicated in the lower panel of Figure 9, which replicates Figure 8. Aligning the two plots allows us to easily indicate which region (in terms of Figure 8) the buyer finds herself in, for any choice of  $\hat{m}$ , given the value of  $\hat{a} = \hat{a}_j$ ,  $j = 1, 2$ . The demand for money is, in general, higher under  $\hat{a} = \hat{a}_1$ , because a buyer who holds less assets typically relies more heavily on her own money holdings. In the region where  $\hat{m} > \bar{m}^{1-5}$ ,  $D_m^1$  and  $D_m^2$  coincide because, for either  $\hat{a} = \hat{a}_1$  or  $\hat{a} = \hat{a}_2$ , the buyer is in Region 1, and the marginal benefit of carrying one more unit of money is independent of  $\hat{a}$ . In particular, in Region 1, one additional unit of money: a) serves as a store of value, if the buyer is an N-type; b) allows the buyer to purchase more goods in the DM, if she is an unmatched C-type; and c) allows the buyer to reduce her demand for the N-type's money, if she is a matched C-type.

As  $\varphi/(\beta \hat{\varphi})$  increases further, the buyer finds herself in Region 5 (if  $\hat{a} = \hat{a}_1$ ) or Region 2 (if  $\hat{a} = \hat{a}_2$ ). In both cases,  $D_m^i$  is continuous, but it is characterized by a kink, and the slope of  $D_m^i$  (in absolute value) is higher on the left side of that kink. To illustrate this property, let  $\hat{a} = \hat{a}_2$  and consider  $D_m^2$  in a neighborhood of  $\bar{m}^{1-2}$ , i.e. consider how the marginal benefit of carrying one additional unit of money changes, as the buyer moves from Region 1 to Region 2. As pointed out earlier, in Region 1 an additional unit of money has three effects. The ones indicated by (a) (store of value when N-type) and (b) (higher marginal utility when unmatched C-type) are still

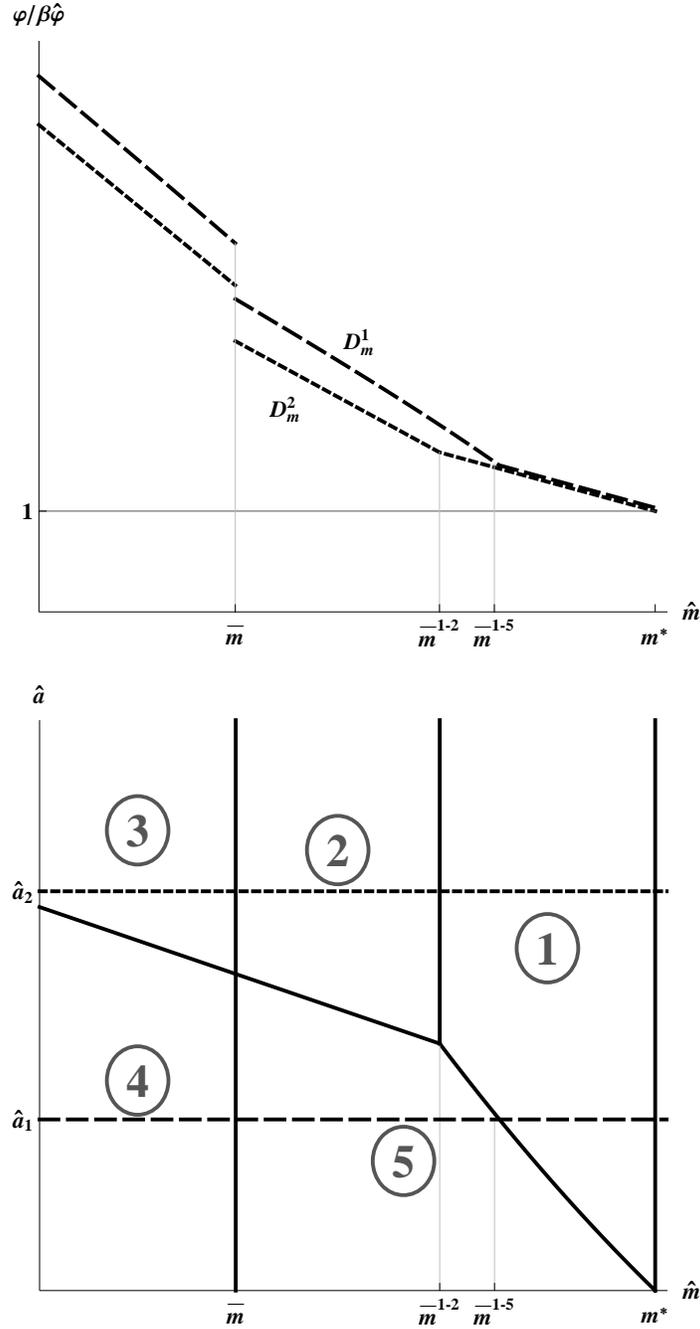


Figure 9: Money demand given high ( $\hat{a}_2$ ) and low ( $\hat{a}_1$ ) asset holdings.

relevant as we pass into Region 2. But now, when the buyer is a matched C-type, one extra unit of money does not just allow her to lower her demand for the N-type's money (effect (c) above), but, in fact, it allows her to increase her purchasing power in the forthcoming DM. Hence, for all  $\hat{m} \in [\bar{m}, \bar{m}^{1-2})$ , the demand curve is steeper in comparison to the range  $[\bar{m}^{1-2}, \infty)$ . Moreover, notice from (a.1) and (a.2) that

$$J_1^2 - J_1^1 = \beta\hat{\varphi}\lambda f \{u' [\beta\hat{\varphi}(\hat{m} + \tilde{m})] - 1\}.$$

This is precisely the term that differentiates  $D_m^2$  on the two sides of  $\bar{m}^{1-2}$ . Clearly, this term goes to zero when  $\hat{m} = \bar{m}^{1-2} = m^* - \tilde{m}$ . As a result,  $D_m^2$  exhibits a kink *but not a jump* at  $\bar{m}^{1-2}$ . The behavior of  $D_m^1$  around  $\bar{m}^{1-5}$  admits a similar interpretation.

Finally, the  $D_m^i$  functions exhibit a discontinuity, or a jump, at  $\bar{m}$ . To illustrate this property, again let  $\hat{a} = \hat{a}_2$  and focus on the behavior of  $D_m^2$  in a neighborhood of  $\bar{m}$  (the behavior of  $D_m^1$  around that point can be explained in a similar fashion). Recall from the discussion above that, in Region 2, an additional unit of money serves as a store of value, if the buyer is an N-type, and it allows the buyer to purchase more goods in the DM, if she is a C-type (matched or unmatched). These effects are also valid as we cross into Region 3. However, Region 3 is also characterized by a whole new effect, which becomes relevant when the buyer is a matched N-type. In Region 3, the C-type counterparty can afford to buy all of the buyer's money, hence the buyer's choice of  $\hat{m}$  affects the OTC terms of trade (to the extent that the N-type has some bargaining power, i.e.  $\lambda < 1$ ). More precisely, the less money the buyer brings, the more desperate the C-type will be for that money and the more assets she will be willing to give up in order to acquire it (i.e.  $\psi_I$  will be very low). In mathematical terms, (a.2) and (a.3) imply that

$$J_1^3 - J_1^2 = \beta \hat{\varphi} (1 - \lambda) f \{ u' [\beta \hat{\varphi} (\hat{m} + \tilde{m})] - 1 \}.$$

Since  $\bar{m} < m^* - \tilde{m}$ , this term is strictly positive when  $\hat{m} = \bar{m}$ , provided that  $\lambda < 1$ . This gap between the values of  $J_1^3$  and  $J_1^2$  is precisely what leads to the jump of  $D_m^2$  at  $\bar{m}$ .

## A.2 Proofs of Statements

*Proof.* Proof of Lemma 1.

The proof is standard. Because of his linear cost, a seller's supply curve is vertical at  $p = 1/(\beta \hat{\varphi})$ , which determines the competitive price. Plugging this into the buyer's problem yields:

$$\max_q \{ u(q) - q \},$$

subject to  $q \leq \beta \hat{\varphi} m$ . As long as  $m \geq m^*$ ,  $q$  will be equal to the first-best quantity, i.e.  $q = q^*$ . On the other hand, if  $m < m^*$ , the cash constraint is binding. The buyer will give up all her money and purchase whichever quantity of goods her money can buy. In other words,  $q = \beta \hat{\varphi} m$ .  $\square$

*Proof.* Proof of Lemma 2.

First, we define the bargaining problem. We are considering a meeting in the OTC market

between a C-type and an N-type with portfolios  $(m, a)$  and  $(\tilde{m}, \tilde{a})$ , respectively.

$$\begin{aligned} & \max_{\chi, \psi_I} \{V^B(m + \chi\psi_I, a - \chi) - V^B(m, a)\} \\ \text{s.t. } & V^B(m + \chi\psi_I, a - \chi) - V^B(m, a) = \frac{\lambda}{1 - \lambda} [\beta W^B(\tilde{m} - \chi\psi_I, \tilde{a} + \chi) - \beta W^B(\tilde{m}, \tilde{a})], \end{aligned}$$

and the cash and asset constraints  $\chi \in [-\tilde{a}, a]$ , and  $\chi\psi_I \in [-m, \tilde{m}]$ .

With proportional bargaining, the objective is to maximize the C-type's surplus, subject to the constraint that the N-type's payment equals a fixed proportion of that surplus (specifically,  $(1 - \lambda)/\lambda$ ). Since the C-type has a consumption opportunity, she will proceed to the DM with an additional  $\chi\psi_I$  units of money, but also with asset holdings reduced by the amount  $\chi$ . On the other hand, the N-type will proceed directly to the next period's CM. If one substitutes the value functions  $W^B, V^B$  from (2) and (7) into the expression above, the bargaining problem can be re-written as

$$\max_{\chi, \psi_I} \{u[q(m + \chi\psi_I)] - u[q(m)] + \beta[\hat{\varphi}\chi\psi_I + \hat{\varphi}pq(m) - \hat{\varphi}pq(m + \chi\psi_I) - d\chi]\} \quad (\text{a.8})$$

subject to:

$$u[q(m + \chi\psi_I)] - u[q(m)] + \beta[\hat{\varphi}\chi\psi_I + \hat{\varphi}pq(m) - \hat{\varphi}pq(m + \chi\psi_I) - d\chi] = \frac{\beta\lambda}{1 - \lambda} (d\chi - \hat{\varphi}\chi\psi_I) \quad (\text{a.9})$$

and  $\chi \in [-\tilde{a}, a]$ ,  $\chi\psi_I \in [-m, \tilde{m}]$ . The expression  $q(\cdot)$  refers to the competitive DM solution described earlier, and  $pq(\cdot)$  is short for  $p(\cdot)q(\cdot)$  which makes for easier reading.

Notice that if we solve equation (a.9) with respect to  $\beta d\chi$ , we obtain

$$\beta d\chi = \beta\hat{\varphi}\chi\psi_I + (1 - \lambda) \{u[q(m + \chi\psi_I)] - u[q(m)] + \beta\hat{\varphi}[pq(m) - pq(m + \chi\psi_I)]\}. \quad (\text{a.10})$$

This expression admits a more intuitive interpretation than its original counterpart. It states that the N-type should receive an amount of assets whose discounted value equals the real discounted value of the money she is giving up (the term  $\beta\hat{\varphi}\chi\psi_I$ ), plus a fraction  $1 - \lambda$  of the net surplus created when a monetary transfer of  $\chi\psi_I$  is made to the C-type (the term in curly brackets). Substituting the term  $\beta d\chi$  from (a.10) into (a.8), simplifies the bargaining problem to

$$\max_{\chi, \psi_I} \lambda \{u[q(m + \chi\psi_I)] - u[q(m)] + \beta\hat{\varphi}[pq(m) - pq(m + \chi\psi_I)]\},$$

subject to the constraint in (a.9), and the feasibility constraints  $\chi \in [-\tilde{a}, a]$ , and  $\chi\psi_I \in [-m, \tilde{m}]$ . As is standard with proportional bargaining, the C-type's surplus turns out to be equal to a fraction  $\lambda$  (her bargaining power) of the total surplus generated when the N-type transfers to the C-type  $\chi\psi_I$  units of money in return to the  $\chi$  units of the asset.

Now we can turn to describing the solution of the bargaining problem. We assume that trade takes place only if a strictly positive surplus is generated. For example, consider the case in which a C-type carries  $m \geq m^*$ . This agent can buy  $q^*$  in the DM, and she will also be able to buy  $q^*$  if she receives more money from the N-type or if she gives up some amount of money up to  $m - m^*$ . In all these cases, the two types are just swapping money for assets, but no surplus is generated. These trivial trades will never affect the equilibrium objects, hence, for simplicity, we rule them out. This, in turn, implies that the only relevant restrictions are  $\chi \leq a$  and  $\chi\psi_I \leq \tilde{m}$ , i.e. the C-type will always be a seller of the asset.

From now on, focus on the case where  $m, \tilde{m} \leq m^*$ , which we know will be true in equilibrium, since  $\mu > \beta - 1$ . Since  $m \leq m^*$ , and we have ruled out swapping of money with assets that does not create surplus, we know that the money holdings of the C-type as she enters the DM will never exceed  $m^*$ . This implies that we will always be on the branch of the DM solution in Lemma 1 where the cash constraint binds, i.e.  $q(m) = \beta\hat{\varphi}m$ . These observations allow us to re-write the bargaining problem as

$$\max_{\chi, \psi_I} \lambda \{u[\beta\hat{\varphi}(m + \chi\psi_I)] - u(\beta\hat{\varphi}m) - \beta\hat{\varphi}\chi\psi_I\},$$

subject to  $\chi \leq a$ ,  $\chi\psi_I \leq \tilde{m}$ , and (a.9). The objective function above depends only on the money,  $\chi\psi_I$ , that changes hands, and the only way through which  $\chi, \psi_I$  are individually relevant is through the feasibility constraints. Defining this objective function as  $G(\chi\psi_I)$ , we have

$$\frac{dG}{d(\chi\psi_I)} = \lambda\beta\hat{\varphi}\{u'[\beta\hat{\varphi}(m + \chi\psi_I)] - 1\} \geq 0,$$

with strict equality for all  $\chi\psi_I < m^* - m$ .

The discussion above suggests that the solution to the bargaining problem follows a simple algorithm. First, one needs to ask whether the N-type carries enough money such that  $m + \tilde{m} \geq m^*$ . In other words, whether if the two types pool all their money together, the C-type can purchase  $q^*$  in the DM. Second, one needs to worry about whether the C-type has enough assets to compensate the N-type for the transfer of liquidity. If  $a$  was unlimited, we would always have  $\chi\psi_I = \min\{m^* - m, \tilde{m}\}$ , but the asset constraint may bind. More formally, consider the following cases.

Case 1:  $m + \tilde{m} \geq m^*$ . In this case, if the asset constraint is not binding, we have  $\chi\psi_I = m^* - m$ . The critical level of assets that the C-type should carry in order to be able to compensate the N-type for  $m^* - m$  units of money, can be found by substituting  $\chi\psi_I = m^* - m$  in (a.9), and solving with respect to  $\chi$ . If we label this critical level of assets as  $\bar{a}(m, \tilde{m})$ , we have

$$\bar{a}(m, \tilde{m}) = \frac{1}{\beta d} \{(1 - \lambda) [u(\beta\hat{\varphi}m^*) - u(\beta\hat{\varphi}m)] + \lambda\beta\hat{\varphi}(m^* - m)\}.$$

If  $a \geq \bar{a}(m, \tilde{m})$ , the asset constraint does not bind, and we have  $\chi = \bar{a}(m, \tilde{m})$  and  $\chi\psi_I = m^* - m$ , implying that  $\psi_I = (m^* - m)/\bar{a}(m, \tilde{m})$ . On the other hand, if  $a < \bar{a}(m, \tilde{m})$ , the C-type cannot acquire the first-best level of money. In this case, she will give up all her assets,  $\chi = a$ , and she will pay a price per unit of asset,  $\psi_I^a(m, a)$ , which implicitly solves (a.9), after substituting  $\chi = a$ . This is just equation (10) in Lemma 2.

Case 2:  $m + \tilde{m} < m^*$ . In this case, if the asset constraint is not binding, we have  $\chi\psi_I = \tilde{m}$ . The critical level of assets that the C-type should carry in order to afford  $\tilde{m}$  can be found by substituting  $\chi\psi_I = \tilde{m}$  in (a.9), and solving with respect to  $\chi$ . This yields

$$\bar{a}(m, \tilde{m}) = \frac{1}{\beta d} \left\{ (1 - \lambda) \left\{ u[\beta\hat{\varphi}(m + \tilde{m})] - u(\beta\hat{\varphi}m) \right\} + \lambda\beta\hat{\varphi}\tilde{m} \right\}.$$

If  $a \geq \bar{a}(m, \tilde{m})$ , the asset constraint does not bind, hence  $\chi = \bar{a}(m, \tilde{m})$ , and  $\chi\psi_I = \tilde{m}$ . If, on the other hand,  $a < \bar{a}(m, \tilde{m})$ , the C-type gives away all her assets,  $\chi = a$ , at the per unit price  $\psi_I^a(m, a)$ , implicitly defined by (10).

It is interesting to see how the solutions  $\chi$  and  $\psi_I$  depend on the bargaining power parameter,  $\lambda$ , and how the per unit price  $\psi_I$  depends on the C-type's money holdings. Here, we focus on the case where  $m + \tilde{m} \geq m^*$ , and  $a$  is plentiful, i.e.  $a \geq \bar{a}(m, \tilde{m})$ , but similar arguments can be used to study comparative statics in the remaining cases, and the results remain unaltered. Under this specification, the amount of assets traded in the OTC match is given by

$$\chi = \frac{(1 - \lambda) [u(q^*) - u(\beta\hat{\varphi}m)] + \lambda(q^* - \beta\hat{\varphi}m)}{\beta d},$$

and the derivative of this expression with respect to  $\lambda$  is

$$\frac{\partial \chi}{\partial \lambda} = \frac{[u(\beta\hat{\varphi}m) - \beta\hat{\varphi}m] - [u(q^*) - q^*]}{\beta d} < 0.$$

The last inequality follows from the facts that  $\beta\hat{\varphi}m < \beta\hat{\varphi}m^* = q^*$ , and the function  $u(q) - q$  is maximized at  $q = q^*$ .

Next, consider the OTC price of the asset. When  $m + \tilde{m} \geq m^*$  and  $a \geq \bar{a}(m, \tilde{m})$ , we have

$$\psi_I = \frac{m^* - m}{(1 - \lambda) [u(q^*) - u(\beta\hat{\varphi}m)] + \lambda(q^* - \beta\hat{\varphi}m)} \beta d.$$

The derivative of this expression with respect to  $\lambda$  is

$$\frac{\partial \psi_I}{\partial \lambda} = \frac{[u(q^*) - q^*] - [u(\beta\hat{\varphi}m) - \beta\hat{\varphi}m]}{\{(1 - \lambda) [u(q^*) - u(\beta\hat{\varphi}m)] + \lambda(q^* - \beta\hat{\varphi}m)\}^2} \beta d > 0.$$

As expected, when the bargaining power of the C-type goes up, the price that she obtains for every unit of asset sold, also increases. Notice that, when  $\lambda = 1$ ,  $\psi_I = d/\hat{\varphi}$ , or equivalently,

$\hat{\varphi}\psi_I = d$ . In words, when the C-type makes take-it-or-leave-it offers to the N-type, the price that she will request, in real discounted terms, equals the whole dividend of the asset, leaving no surplus to the N-type.

Finally, we have

$$\frac{\partial\psi_I}{\partial m} = \frac{u'(\beta\hat{\varphi}m)(q^* - \beta\hat{\varphi}m) - [u(q^*) - u(\beta\hat{\varphi}m)]}{\{(1 - \lambda)[u(q^*) - u(\beta\hat{\varphi}m)] + \lambda(q^* - \beta\hat{\varphi}m)\}^2} (1 - \lambda)\beta d > 0. \quad (\text{a.11})$$

The sign of  $\partial\psi_I/\partial m$  coincides with the sign of the numerator on the right-hand side of (a.11), and we show that this is positive for all  $m \in [0, m^*)$ . To see this, define the numerator in (a.11) as  $N(m)$ . Notice the following three facts: First,  $N(0) = \infty$ . Second,  $\lim_{m \rightarrow m^*} N(m) = 0$ . Third,  $N'(m) = \beta\hat{\varphi}u''(\beta\hat{\varphi}m)(q^* - \beta\hat{\varphi}m) < 0$ , for all  $m < m^*$ . Summing up these three observations verifies that  $N(m) > 0$  for all  $m \in [0, m^*)$ , which implies  $\partial\psi_I/\partial m > 0$ .  $\square$

*Proof.* Proof of Lemma 3.

The equilibrium objects  $q_1, q_2, \chi$  and  $\psi_I$  are all deterministic functions of  $z$ , so it is sufficient to focus on  $z$  and  $\psi$  alone. Since  $\mu > \beta - 1$ , we have  $\varphi > \beta\hat{\varphi}$ , and, consequently, parts (b) and (c) of Lemma 6 apply, and an optimal  $(\hat{m}, \hat{a})$  exists and  $\hat{m}$  is unique. The object  $\hat{\varphi}$  (and a proportional  $\varphi = (1 + \mu)\hat{\varphi}$ ) must be chosen such that  $\hat{m} = (1 + \mu)M$ . Finally,  $z = (1 + \mu)\hat{\varphi}M$ , so  $z$  exists and is unique, and so are the equilibrium objects  $q_1, q_2, \chi$  and  $\psi_I$ . But so far they are still functions of  $\psi$ .

Finally, set  $\hat{a} = A$ . Examine the asset demand function (equations (a.6) and (a.7)). It is constant in Regions 1 and 3 strictly decreasing in  $\hat{a}$  in Region 5 (also see the proof of Lemma 5, part iv.), and has a jump at the point where  $(z, A)$  is on the aggregate boundary 5-3. If  $(z, A)$  lies in the interior of Region 5, then  $\psi > \beta d$  is unique. If  $(z, A)$  lies in the interior of Regions 1 or 3, or on the aggregate boundary 1-3, then  $\psi = \beta d$  is unique. But if in equilibrium  $(z, A)$  is on the aggregate boundary 5-3, then  $\psi$  is not unique. Both first-order conditions have a jump, and multiple combinations of  $(\mu, \psi)$  are consistent with the same point  $(z, A)$ .  $\square$

*Proof.* Proof of Lemma 4.

The asset supply cutoff point  $\bar{A}$  is just given by  $\bar{a}(m^*/2, m^*/2)$ , and it represents the highest level of asset supply that is consistent with aggregate Region 5 (see Figure 1).

Case 1: Let  $A \geq \bar{A}$ . Any equilibrium must satisfy  $\hat{a} = A$  for all agents, whether they turn out to be C-types or N-types. By the definition of the regions, we must be in aggregate Regions 1 or 3. Now money demand (equations (a.1) and (a.3)) is continuous within each region, and also continuous across the boundary 1-3 (because the boundary is defined by  $\hat{m} + \tilde{m} = m^*$ , hence  $u'(\hat{m} + \tilde{m}) = 1$ ), and finally strictly decreasing over the entire range. Therefore, there exists a  $\tilde{\mu} > \beta - 1$  such that the statement holds.

Case 2: Let  $A < \bar{A}$ . Any equilibrium must satisfy  $\hat{a} = A$  for all agents, whether they turn

out to be C-types or N-types. By the definition of the aggregate regions, we could be in Regions 1 (high  $z$ ), 5 (intermediate  $z$ ), or 3 (low  $z$ ). Money demand (equations (a.1), (a.5), and (a.3)) is continuous within Regions 1 and 5, and across the boundary 1-5, and strictly decreasing within both regions. Therefore, there exists a  $\mu' > \beta - 1$  as stated in the lemma.

Next, consider a candidate equilibrium  $(z, A)$  on the boundary 3-5. By the buyer's optimal behavior,  $J_1^5 \leq 0$  and  $J_1^3 \geq 0$  at this point. Both  $J_1^5$  and  $J_1^3$  are strictly decreasing in  $\mu$  for a fixed  $(z, A)$  (and a fixed  $\varphi$ ). Let  $\mu''$  denote the lowest  $\mu$  consistent with  $J_1^5 \leq 0$ , and  $\mu'''$  the highest  $\mu$  consistent with  $J_1^3 \geq 0$ . As shown in the proof of Lemma 5,  $J_1^5 < J_1^3$  at that boundary, so it must be that  $\mu'' < \mu'''$ . Finally,  $\mu' < \mu''$  follows from the fact that money demand is strictly decreasing within Region 5.  $\square$

*Proof.* Proof of Proposition 1.

Case 1: Let  $A \geq \bar{A}$ . Then the equilibrium can only be in the aggregate Regions 1 or 3. By equation (a.6), the only solution to  $J_2 = 0$  in Regions 1 and 3 is  $\psi = \beta d$ .

Case 2: Let  $A < \bar{A}$ . By Lemma 4, the equilibrium is in Regions 1, 3, or 5, or on the boundary of Regions 5 and 3, for the according money growth rate  $\mu$ .

a) The range  $\mu \in (\beta - 1, \mu']$  corresponds to Region 1, and  $\mu \in (\mu''', \infty)$  corresponds to Region 3. As shown for Case 1,  $\psi = \beta d$  in these regions.

b) The range  $\mu \in (\mu', \mu'')$  corresponds to Region 5. Asset demand in Region 5 satisfies equation (a.7). As the argument of  $u' [\beta \hat{\varphi}(\hat{m} + \hat{a} \psi_I^a(\hat{m}, \hat{a}))]$  is less than  $m^*$  by the definition of the region,  $\psi > \beta d$ . Furthermore, higher inflation reduces  $\hat{m}$ . The term  $\psi_I^a(\hat{m}, \hat{a})$  is itself an increasing function of  $\hat{m}$ , as shown in equation (a.11), so higher inflation reduces the argument and increases the term  $u' [\beta \hat{\varphi}(\hat{m} + \hat{a} \psi_I^a(\hat{m}, \hat{a}))]$ , and also increases  $\psi$ . Equation (13) follows from imposing the equilibrium conditions on equation (a.7).

c) The range  $\mu \in [\mu'', \mu''']$  corresponds to the boundary of Regions 5 and 3, where the objective function has a kink. Clearly,  $\psi$  still cannot be lower than  $\beta d$ . The upper limit of  $\psi$  is given by the  $\psi(\mu'')$ , which is the limit of  $\psi(\mu)$  in Region 5, because any  $\psi > \psi(\mu'')$  would lead the agent to demand less assets than the point on the boundary, and equilibrium would move into the interior of Region 5.

Finally, consider the OTC (real) asset price,  $\varphi \psi_I$ . This term can increase or decrease with inflation, and this depends on  $\ell$ ,  $\lambda$ , and the shape of the utility function. (The exception is the boundary 5-3, i.e.  $\mu \in [\mu'', \mu''']$ . There,  $\varphi \psi_I$  must be increasing, because  $z$  is constant and the only thing that matters is that the value of money falls with higher  $\mu$ .) An example is illustrated in Figure 5. Equation (14) follows from imposing the equilibrium conditions on the bargaining rule in equation (a.10).  $\square$

*Proof.* Proof of Proposition 2.

Case 1: Let  $A \geq \bar{A}$ . Then the equilibrium can only be in the aggregate Regions 1 or 3. By

equations (a.1) and (a.3), and using  $\hat{m} = M$ ,  $z = \varphi M = \hat{\varphi}M/(1 + \mu)$ , and  $\varphi = (1 + \mu)\hat{\varphi}$ , demand for real balances  $z$  is strictly decreasing as a function of  $\mu$ . The average DM production is  $q^{DM} \equiv fq_2 + (\ell - f)q_1$ . By Lemma 4, equilibrium is in Region 1 if  $\mu < \tilde{\mu}$  and in Region 3 otherwise. In Region 1,  $q_2 = q^*$  and  $q_1 = \beta z$ , so  $q^{DM}$  falls if  $z$  does unless  $f = \ell$ . In Region 3,  $q_2 = 2\beta z$  and  $q_1 = \beta z$ , so  $q^{DM}$  falls if  $z$  does.

Case 2: Let  $A < \bar{A}$ . By Lemma 4, the equilibrium is in Regions 1, 3, or 5, or on the boundary of Regions 5 and 3, for the according money growth rate  $\mu$ .

a,b) The proof is the same as for Case 1, with the addition of noting that  $\hat{m}$  (and hence  $z$ ) is a strictly decreasing function of inflation in Region 5 as well.

c) The boundary of Regions 5 and 3, together with the fact that  $A$  is given, fixes  $z$  and hence  $q^{DM} = f \cdot 2\beta z + (\ell - f) \cdot \beta z$ .  $\square$

*Proof.* Proof of Proposition 3.

Case 1: Let  $A \geq \bar{A}$ . Then the equilibrium can only be in the aggregate Regions 1 or 3. The measure of matched agents in the OTC,  $f$ , is constant, and  $\chi = \bar{a}(\hat{m}, \tilde{m})$  in Regions 1 and 3. So in aggregate Region 1,  $\chi = \{(1 - \lambda)[u(q^*) - u(\beta z)] + \lambda(q^* - \beta z)\}/(\beta d)^{-1}$ , which is strictly decreasing in  $z$ , therefore  $\chi$  is strictly increasing as a function of  $\mu$ . In aggregate Region 3, on the other hand,  $\chi = \{(1 - \lambda)[u(2\beta z) - u(\beta z)] + \lambda\beta z\}/(\beta d)^{-1}$ , which is strictly increasing in  $z$ , therefore  $\chi$  is strictly decreasing as a function of  $\mu$ .

Case 2: Let  $A < \bar{A}$ . By Lemma 4, the equilibrium is in Regions 1, 3, or 5, or on the boundary of Regions 5 and 3, for the according money growth rate  $\mu$ . As in Case 1,  $f$  is constant. That  $\chi$  is increasing in Region 1 and decreasing in Region 3 was proven for Case 1. In Region 5 and on the boundary 5-3, the OTC asset constraint is binding, implying that  $\chi = A$ , which is constant.  $\square$

*Proof.* Proof of Proposition 4.

Refer to the Web Appendix for the statement and solution of the competitive model. We show that for any  $\mu > \beta - 1$ , DM consumption  $q$  satisfies  $q < q^*$ .

Now consider the model with the OTC secondary asset market.  $\ell > 0$  and  $\lambda < 1$  guarantee that Region 1 in  $(\mu, A)$ -space (see Figure 2) is non-empty. If  $\mu \leq \mu'$  given that  $A < \bar{A}$ , or if  $\mu \leq \tilde{\mu}$  given that  $A \geq \bar{A}$ , then the equilibrium is in Region 1, and matched C-types achieve the first-best level of consumption  $q = q^*$ . Finally, if  $f = \ell$  (in which case we need  $\ell \leq 1 - \ell$ ), then all C-types match in the OTC secondary market. Consequently, DM production is higher in the model where the secondary market is OTC, as long as inflation is not too large.  $\square$